SECTION B

JOURNAL OF RESEARCH

of the NATIONAL BUREAU OF STANDARDS

JNBBAU 80B(4) 433-509 (Oct.-Dec. 1976)

U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

Mathematical Sciences

Journal of Research

of the

National Bureau of Standards

B. Mathematical Sciences

Oct.–Dec. • 1976

VOLUME • 80B

NUMBER • 4

Editor: Morris Newman

Associate Editor: F. W. Olver

Publication dates: Mar. 10, June 10, Sept. 10, Dec. 10, 1976

J. Res. Nat. Bur. Stand. (U.S.), B

JOURNAL OF RESEARCH

The National Bureau of Standards Journal of Research reports research and development in the fields of activity shown at right. Also included from time to time are survey articles on topics closely related to the Bureau's scientific and technical program.

The Journal is published in two separate sections as follows:

A. Physics and Chemistry

Contains papers of interest primarily to scientists working in these fields. Issued six times a year. Annual subscription: domestic, $17.00; foreign, $21.25. Single copy, varies.

B. Mathematical Sciences

Presents studies and compilations designed mainly for the mathematician and the theoretical physicist. Issued quarterly. Annual subscription: domestic, $9.00; foreign, $11.25. Single copy $2.25.

NATIONAL BUREAU OF STANDARDS

The National Bureau of Standards[1] was established by an act of Congress March 3, 1901. The Bureau's overall goal is to strengthen and advance the Nation's science and technology and facilitate their effective application for public benefit. To this end, the Bureau conducts research and provides: (1) a basis for the Nation's physical measurement system, (2) scientific and technological services for industry and government, (3) a technical basis for equity in trade, and (4) technical services to promote public safety. The Bureau consists of the Institute for Basic Standards, the Institute for Materials Research, the Institute for Applied Technology, the Institute for Computer Sciences and Technology, the Office for Information Programs, and the Office of Experimental Technology Incentives Program.

THE INSTITUTE FOR BASIC STANDARDS provides the central basis within the United States of a complete and consistent system of physical measurement; coordinates that system with measurement systems of other nations; and furnishes essential services leading to accurate and uniform physical measurements throughout the Nation's scientific community, industry, and commerce. The Institute consists of the Office of Measurement Services, and the following center and divisions:

Applied Mathematics — Electricity — Mechanics — Heat — Optical Physics — Center for Radiation Research — Laboratory Astrophysics[2] — Cryogenics[2] — Electromagnetics[2] — Time and Frequency[2].

THE INSTITUTE FOR MATERIALS RESEARCH conducts materials research leading to improved methods of measurement, standards, and data on the properties of well-characterized materials needed by industry, commerce, educational institutions, and Government; provides advisory and research services to other Government agencies; and develops, produces, and distributes standard reference materials. The Institute consists of the Office of Standard Reference Materials, the Office of Air and Water Measurement, and the following divisions:

Analytical Chemistry — Polymers — Metallurgy — Inorganic Materials — Reactor Radiation — Physical Chemistry.

THE INSTITUTE FOR APPLIED TECHNOLOGY provides technical services developing and promoting the use of available technology; cooperates with public and private organizations in developing technological standards, codes, and test methods; and provides technical advice services, and information to Government agencies and the public. The Institute consists of the following divisions and centers:

Standards Application and Analysis — Electronic Technology — Center for Consumer Product Technology: Product Systems Analysis; Product Engineering — Center for Building Technology: Structures, Materials, and Safety; Building Environment; Technical Evaluation and Application — Center for Fire Research: Fire Science; Fire Safety Engineering.

THE INSTITUTE FOR COMPUTER SCIENCES AND TECHNOLOGY conducts research and provides technical services designed to aid Government agencies in improving cost effectiveness in the conduct of their programs through the selection, acquisition, and effective utilization of automatic data processing equipment; and serves as the principal focus within the executive branch for the development of Federal standards for automatic data processing equipment, techniques, and computer languages. The Institute consist of the following divisions:

Computer Services — Systems and Software — Computer Systems Engineering — Information Technology.

THE OFFICE OF EXPERIMENTAL TECHNOLOGY INCENTIVES PROGRAM seeks to affect public policy and process to facilitate technological change in the private sector by examining and experimenting with Government policies and practices in order to identify and remove Government-related barriers and to correct inherent market imperfections that impede the innovation process.

THE OFFICE FOR INFORMATION PROGRAMS promotes optimum dissemination and accessibility of scientific information generated within NBS; promotes the development of the National Standard Reference Data System and a system of information analysis centers dealing with the broader aspects of the National Measurement System; provides appropriate services to ensure that the NBS staff has optimum accessibility to the scientific information of the world. The Office consists of the following organizational units:

Office of Standard Reference Data — Office of Information Activities — Office of Technical Publications — Library — Office of International Standards — Office of International Relations.

[1] Headquarters and Laboratories at Gaithersburg, Maryland, unless otherwise noted; mailing address Washington, D.C. 20234.
[2] Located at Boulder, Colorado 80302.

U.S. DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary
NATIONAL BUREAU OF STANDARDS
Ernest Ambler, Director

Order all publications from the Superintendent of Documents
U.S. Government Printing Office, Washington, D.C. 20402.

The Secretary of Commerce has determined that the publication of this periodical is necessary in the transaction of the public business required by law of this Department. Use of funds for printing this periodical has been approved by the Director of the Office of Management and Budget through June 30, 1976.

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematical Sciences
Vol. 80B, No. 4, October–December 1976

An Inequality for Doubly Stochastic Matrices*

Charles R. Johnson and R. Bruce Kellogg****

Institute for Basic Standards, National Bureau of Standards, Washington, D.C. 20234

(June 30, 1976)

Interrelated inequalities involving doubly stochastic matrices are presented. For example, if B is an n by n doubly stochastic matrix, x any nonnegative vector and $y = Bx$, then $x_1x_2\cdots x_n \leq y_1y_2\cdots y_n$. Also, if A is an n by n *nonnegative matrix and* D and E are positive diagonal matrices such that $B = DAE$ is doubly stochastic, then $\det DE \geq \rho(A)^{-n}$, where $\rho(A)$ is the Perron-Frobenius eigenvalue of A. The relationship between these two inequalities is exhibited.

Key words: Diagonal scaling; doubly stochastic matrix; Perron-Frobenius eigenvalue.

An n by n entry-wise nonnegative matrix $B = (b_{ij})$ is called row (column) *stochastic* if $\sum_{j=1}^{n} b_{ij} = 1$ for all $i = 1,\cdots,n$ $\left(\sum_{i=1}^{n} b_{ij} = 1 \text{ for all } j = 1,\cdots,n\right)$. If B is simultaneously row and column stochastic then B is said to be *doubly stochastic*. We shall denote the Perron-Frobenius (maximal) eigenvalue of an arbitrary n by n entry-wise nonnegative matrix A by $\rho(A)$. Of course, if A is stochastic, $\rho(A) = 1$.

It is known precisely which n by n nonnegative matrices may be diagonally scaled by positive diagonal matrices D, E so that

$$B = DAE \tag{1}$$

is doubly stochastic. If there is such a pair D, E, we shall say that A has property (*). In this event it is our interest to obtain inequalities on D and E. In the process, certain related inequalities for doubly stochastic matrices are noticed.

It was first realized by Sinkhorn [4][1] that if A is entry-wise positive and square, then A has property (*). The proof amounts to showing that the process of alternately scaling A to produce a row stochastic matrix, and then a column stochastic matrix, and then continuing the process, actually converges to a doubly stochastic matrix. The hypothesis of positivity, however, can be weakened somewhat. If there exists no single permutation matrix P such that

$$P^TAP = \begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix}$$

where A_{11} and A_{22} are square, then A is called irreducible. If there is a pair of permutation matrices P, Q such that $C = PAQ$, then we shall say that A and C are *equivalent*. If, further, A is equivalent to no matrix of the form

AMS Subject Classification: 15A45, 15A48, 15A51, 65F35.
* Work supported in part by NSF grant GP20555.
** Present address: Institute for Physical Science and Technology, University of Maryland, College Park, Md. 20742.
[1] Figures in brackets indicate the literature references at the end of this paper.

(2)
$$\begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix}$$

where A_{11} and A_{22} are square, then A is termed *completely irreducible.* It is an easy calculation to show that A has property (*) if and only if each matrix equivalent to A does, and it is equally clear that for A to have property (*) it must have the zero-nonzero sign pattern of a doubly stochastic matrix. For example, if A has property (*) and if A is of form (2) this means we must then have $A_{21} = 0$. It has further been shown [1] that when A is completely irreducible the alternate scaling process of Sinkhorn still converges and thus A has property (*). Since property (*) is preserved under direct summation, we may summarize as follows.

REMARK 1: *A square nonnegative matrix* A *has property* (*) *if and only if* A *is completely irreducible or* A *is equivalent to a direct sum of completely irreducible matrices.*
Thus, property (*) depends only on the zero pattern of A. It is also a straightforward calculation (following [4]) that

REMARK 2: *If* A *has property* (*), *then the product* DE *of* (1) *is unique.*

Our first observation is both necessary for later proofs and of interest by itself.

THEOREM 1: *If* $B = (b_{ij})$ *is an* n *by* n *doubly stochastic matrix and* $x \geq 0$ *is any nonnegative vector, then, for* $y = Bx$, *we have*

(3)
$$\prod_{i=1}^{n} x_i \leq \prod_{i=1}^{n} y_i.$$

If B *is completely irreducible, equality holds in* (3) *if and only if the right-hand side is* 0 *or all components of* x *are the same. Furthermore, among all irreducible nonnegative square matrices* B *satisfying* $\rho(B) \leq 1$, *only those diagonally similar to doubly stochastic matrices satisfy* (3) *for all* $x \geq 0$.

PROOF: From the arithmetic-geometric mean inequality [2]

(4)
$$\prod_{i=1}^{n} x_i^{\gamma_i} \leq \sum_{i=1}^{n} \gamma_i x_i$$

where $x = (x_1, \cdots, x_n)^T$ is any nonnegative vector, and $\gamma = (\gamma_1, \cdots, \gamma_n)$ is a vector of nonnegative numbers satisfying $\sum_{i=1}^{n} \gamma_i = 1$. Equality holds in (4) if and only if the x_i's corresponding to nonzero γ_i's are all equal. Now, suppose $B = (b_{ij})$ is row stochastic and $y = Bx$, $x \geq 0$. It follows from (4) that

(5)
$$\prod_{j=1}^{n} x_j^{b_{ij}} \leq \sum_{j=1}^{n} b_{ij} x_j = y_i, \qquad \text{for } i = 1, \cdots, n.$$

Taking a product over i of both sides, we arrive at

(6)
$$\prod_{j=1}^{n} x_j^{\sum_{i=1}^{n} b_{ij}} \leq \prod_{i=1}^{n} y_i.$$

If B is doubly stochastic, $\sum_{i=1}^{n} b_{ij} = 1$ for each $j = 1, \cdots, n$, and it follows that (3) holds.

To analyze the case of equality, it is clear that equality holds in (3) if either x is a vector of equal components or the right-hand side of (3) is 0. On the other hand, if equality holds in (3) and the right-hand side of (3) is not 0, then equality must hold in (5) for each $i = 1, \cdots, n$. This means that for each i, the x_j's corresponding to nonzero b_{ij}'s are all equal. This, in turn, implies, by virtue of equality holding in (5) for all i, that $y = Q^T x$ for some permutation matrix Q. Since $BQQ^T x = y$, we have that BQ has $Q^T x = y$ as a Perron-Frobenius eigenvector (corresponding to $\rho(BQ) = 1$). If B is completely irreducible, then BQ is irreducible, and, since BQ is doubly stochastic, its Perron-Frobenius eigenspace is one-dimensional (because of the irreducibility) and is spanned by $(1,1,\cdots,1)^T$. Therefore all components of $Q^T x$, and thus of x, are the same. It should be noted that even in case B is completely irreducible it is possible that the right-hand side of (3) be 0 for a nonnegative nonzero vector x. For example, let

$$B = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/6 & 1/6 & 2/3 \\ 1/2 & 1/2 & 0 \end{pmatrix} \text{ and } x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Thus, the second statement of the theorem is proven. It should also be noted that the case of equality in (3) may similarly be analyzed if B is equivalent to a direct sum of completely irreducibles. It is enough to assume B is equal to a direct sum of completely irreducibles. Then, in addition to equality occurring in (3) when the right-hand side is 0, equality occurs precisely when the components of x are equal within each piece corresponding to a direct summand of B.

For the third statement, it is enough to assume that B satisfies $\rho(B) = 1$. It then follows from the irreducibility of B that there is a positive diagonal matrix D such that DBD^{-1} is row stochastic (D^{-1} is obtained from the Perron-Frobenius eigenvector of B, which is positive). Since $Bx = y$ if and only if $(DBD^{-1})Dx = Dy$, (3) holds for DBD^{-1} if it holds for B, so that we may as well assume B is row stochastic. Then, if B is not doubly stochastic, some column sum is <1, so let $\theta = \sum_i b_{ij} < 1$. Let $x_i = 1$, $i \neq j$, and $x_j = 1 + \epsilon$. Then (3) becomes

$$1 + \epsilon \leq \prod_i (1 + b_{ij}\epsilon) = 1 + \theta\epsilon + 0(\epsilon^2),$$

which is impossible if $\epsilon > 0$ is small enough. This completes the proof of the theorem.

It should be noted that essential portions of theorem 1 may also be demonstrated by a maximization argument.

EXAMPLE: The assumption of irreducibility in the third statement of theorem 1 cannot, in general, be relaxed. If $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ then B satisfies (3) for all $x \geq 0$ and $\rho(B) = 1$, but B is not similar to a doubly stochastic matrix.

An alternate form of (3) is

COROLLARY 1: *If* $\mathrm{B} = (\mathrm{b_{ij}})$ *is an* n *by* n *doubly stochastic matrix, then for any* n *real numbers* $\mathrm{t_1}, \cdots, \mathrm{t_n}$, *satisfying* $\mathrm{t_i} \geq -1$, $\mathrm{i} = 1, \cdots, \mathrm{n}$, *we have*

$$\prod_{\mathrm{i}=1}^{\mathrm{n}} (1 + \mathrm{t_i}) \leq \prod_{\mathrm{i}=1}^{\mathrm{n}} (1 + \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{t_j b_{ij}}).$$

Our primary observation concerns row stochastic matrices with property (*).

THEOREM 2: *If* A *is a row stochastic matrix with property* (*) *and* D *and* E *are the positive diagonal matrices guaranteed by* (1) *then*

$$\det \mathrm{DE} \geq 1. \tag{7}$$

Furthermore, equality holds if and only if A *is actually doubly stochastic.*

PROOF: As $B = (b_{ij})$ runs through all n by n doubly stochastic matrices and $F = \operatorname{diag}\{f_1, \cdots, f_n\}$ runs through all positive diagonal matrices, then $A = D^{-1}BF$ runs through all row stochastic matrices with property (*) where $D = \operatorname{diag}\{\sum_{j=1}^{n} b_{1j}f_j, \cdots, \sum_{j=1}^{n} b_{nj}f_j\}$. Thus, since $B = DAE$, where $E = F^{-1}$, it suffices to show that $\det D \geq \det F$. If we denote $(f_1, \cdots, f_n)^T$ by f, this is equivalent to saying that the product of the entries of Bf is greater than or equal to that of f for any positive vector f. This, of course, follows from theorem 1. To analyze the case of equality in (7), it suffices to assume B is completely irreducible. In this event, it follows from theorem 1 and the fact that Bf has no 0 components that equality in (7) implies that all entries of f are the same. Thus $D = F$ and equality holds in (7) precisely when A is already doubly stochastic.

Note: A related but rather different inequality when A is symmetric appears in [3, theorem 3]. Also a portion of the proof of that result could be used to prove part of the first statement in our theorem 1.

It follows from theorem 2 that

COROLLARY 2: *If* A *is a row stochastic matrix with property* (*) *and* B *is related to* A *by* (1), *then* $|\det \mathrm{A}| \leq |\det \mathrm{B}|$.

We denote the eigenvalues of A by $a_1,\cdots,a_n$, ordered so that $|a_1| \leq \cdots \leq |a_n|$, and those of B by $\beta_1,\cdots,\beta_n$, ordered so that $|\beta_1| \leq \cdots \leq |\beta_n|$. Since $a_n = 1 = \beta_n$, it follows from corollary 2 that

COROLLARY 3: *If* A *is a row stochastic matrix with property* (*) *and* B *is related to* A *by* (1), *then* $\prod_{i=1}^{n-1} |a_i| \leq \prod_{i=1}^{n-1} |\beta_i|$.

We conjecture that in case A is row stochastic with property (*) and B is related to A by (1), then actually

$$|a_i| \leq |\beta_i|, \; i = 1,\cdots,n.$$

The result of theorem 2 may be extended to all matrices with property (*) in the following way.

THEOREM 3: *If* A *is any nonnegative* n *by* n *matrix with property* (*) *and* D *and* E *are positive diagonal matrices guaranteed by* (1), *then* $\det DE \geq \rho(A)^{-n}$. *Furthermore, equality holds only if* $DE = \rho(A)^{-1}I$.

PROOF: It is enough to assume A is completely irreducible (for, if not, it is equivalen to a direct sum of same) and then A is irreducible. In this event there is a positive vector x such that $Ax = \rho(A)x$ and, therefore, $\frac{1}{\rho(A)} X^{-1}AX$ is row stochastic, where $X = \operatorname{diag}\{x_1,\cdots,x_n\}$. Application of theorem 2 to $\frac{1}{\rho(A)} X^{-1}AX$ yields $\det D'E' \geq 1$ where $D'(\frac{1}{\rho(A)} X^{-1}AX)E' = B$. Setting $D = \frac{1}{\rho(A)} D'X^{-1}$ and $E = XE'$, gives $B = DAE$ and $\det DE \geq \rho(A)^{-n}$ as was to be shown. The case of equality also follows from theorem 2.

REMARK 3: The reader may wish to note the relationship between the present work and the notion of the *equilibrant*,

$$E(B) \equiv \inf \rho(FB)$$

(where the inf is taken over all positive diagonal matrices of determinant 1), of a nonnegative matrix mentioned in [5].

References

[1.] Brualdi, R., Parter, S., and Schneider, H., The diagonal equivalence of a nonnegative matrix to a stochastic matrix, J. Math. Anal. and Appl. **16**, 31–50 (1966).

[2.] Hardy, G., Littlewood, J., Pólya, G., Inequalities, (Cambridge University Press, 1959).

[3.] Marcus, M., Newman, M., Generalized functions of symmetric matrices, Proc. AMS **16**, 826–830, (1965).

[4.] Sinkhorn, R., A relationship between arbitrary positive matrices, and doubly stochastic matrices, Ann. Math. Stat. **35**, 876–879, (1964).

[5.] Hoffman, A., Linear G-functions, Lin. and Multilin. Alg. **3**, 45–52, (1975).

(Paper 80B4–454)

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematical Sciences
Vol. 80B, No. 4, October–December 1976

Improved Error Bounds for Second-Order Differential Equations With Two Turning Points

F. W. J. Olver*

Institute for Basic Standards, National Bureau of Standards, Washington, D.C. 20234

(July 13, 1976)

New error bounds are given for approximate solutions of differential equations of the form

$$d^2w/d\zeta^2 = \{u^2(\pm\alpha^2-\zeta^2) + \psi(u,\alpha,\zeta)\}w$$

in terms of parabolic cylinder functions. Here ζ is a real variable, α is a bounded real parameter, u is a large positive parameter, and $\psi(u,\alpha,\zeta)$ is a continuous function of α and ζ. Zero values of α are admitted.

Key words: Error bounds; parabolic cylinder functions; turning points; uniform asymptotic approximations; Weber's equation.

1. Introduction and Summary

Recently [2][1], [3], I developed a uniform asymptotic theory of second-order linear differential equations with two coalescing simple turning points, and applied the results to the associated Legendre equation. Subsequently, in the course of writing a paper on connection formulas for multiple turning points [4] it became clear how to effect some improvements in two of the four cases treated in [2]. The purpose of the present note is to describe these improvements. It will be assumed that the reader is familiar with the results presented in [2], and the same notation will be used except where indicated otherwise.

The next section introduces new auxiliary functions for the solutions of the modified Weber equation. The new form of the general approximation theorem is stated and discussed in the third (and concluding) section.

An application of the results to the approximation of Whittaker functions with both parameters large will be published in due course.

2. Auxiliary Functions for the Modified Weber Functions

The modified Weber functions $W(b,x)$ and $W(b,-x)$ are solutions of the differential equation

$$d^2w/dx^2 = (b - \tfrac{1}{4}x^2)w.$$

As in [2], §8.6 we denote by $\sigma(b)$ the smallest positive root of the equation

$$k^{-1/2}(b)W(b,x) = k^{1/2}(b)W(b,-x),$$

* Present address: Institute for Physical Science and Technology, University of Maryland, College Park, Md. 20742.
[1] Figures in brackets indicate the literature references at the end of this paper.

but we now extend this definition to all real values of the parameter b. We already know that

$$\sigma(b) \sim 2b^{1/2}, \qquad b \to +\infty.$$

Next, because $\sigma(b)$ is bounded by the smallest positive zero of $W(b,x)$, the expansion (15.7) of [1] shows that[2]

$$\sigma(b) \to 0, \qquad b \to -\infty.$$

In place of the definition [2], (8.15) of the weight function $E(b,x)$, we now define

$$E(b,x) = \begin{cases} k^{1/2}(b), & -\infty < x \leq -\sigma(b), \\ \{W(b,-x)/W(b,x)\}^{1/2}, & -\sigma(b) \leq x \leq \sigma(b), \\ k^{-1/2}(b), & \sigma(b) \leq x < \infty. \end{cases}$$

Then $E(b,x)$ is continuous for all real values of b and x, and is also a nondecreasing function of x. Furthermore, denoting $1/E(b,x)$ by $E^{-1}(b,x)$, we have

$$E(b,-x) = E^{-1}(b,x).$$

Using the above definition of $E(b,x)$ we now define modulus and phase functions for all real values of b and x by the relations

$$\begin{aligned} W(b,x) &= E^{-1}(b,x)M(b,x)\sin\theta(b,x), \\ W(b,-x) &= E(b,x)M(b,x)\cos\theta(b,x), \\ W'(b,x) &= E^{-1}(b,x)N(b,x)\sin\omega(b,x), \\ W'(b,-x) &= E(b,x)N(b,x)\cos\omega(b,x). \end{aligned}$$

Thus when $-\infty < x \leq -\sigma(b)$ we have

$$\begin{aligned} M(b,x) &= \{k(b)W^2(b,x) + k^{-1}(b)W^2(b,-x)\}^{1/2}, \\ \theta(b,x) &= \tan^{-1}\{k(b)W(b,x)/W(b,-x)\}, \\ N(b,x) &= \{k(b)W'^2(b,x) + k^{-1}(b)W'^2(b,-x)\}^{1/2}, \\ \omega(b,x) &= \tan^{-1}\{k(b)W'(b,x)/W'(b,-x)\}. \end{aligned}$$

Secondly, when $-\sigma(b) \leq x \leq \sigma(b)$ we have

$$\begin{aligned} M(b,x) &= \{2W(b,x)W(b,-x)\}^{1/2}, \qquad \theta(b,x) = \tfrac{1}{4}\pi, \\ N(b,x) &= \left\{\frac{W'^2(b,x)W^2(b,-x) + W'^2(b,-x)W^2(b,x)}{W(b,x)W(b,-x)}\right\}^{1/2}, \\ \omega(b,x) &= \tan^{-1}\left\{\frac{W'(b,x)W(b,-x)}{W'(b,-x)W(b,x)}\right\}. \end{aligned}$$

Lastly, when $\sigma(b) \leq x < \infty$ we have

$$\begin{aligned} M(b,x) &= \{k^{-1}(b)W^2(b,x) + k(b)W^2(b,-x)\}^{1/2}, \\ \theta(b,x) &= \tan^{-1}\{k^{-1}(b)W(b,x)/W(b,-x)\}, \\ N(b,x) &= \{k^{-1}(b)W'^2(b,x) + k(b)W'^2(b,-x)\}^{1/2}, \\ \omega(b,x) &= \tan^{-1}\{k^{-1}(b)W'(b,x)/W'(b,-x)\}. \end{aligned}$$

[2] The actual asymptotic form of $\sigma(b)$ as $b \to -\infty$ can be shown to be $\sigma(b) \sim \frac{1}{2}|b|^{-1/2}e^{\pi b}$, but we shall not make use of this more precise result in the present paper.

Each of the functions $M(b,x)$, $\theta(b,x)$, $N(b,x)$, and $\omega(b,x)$ is continuous in b and x. Reflection properties are given by

$$M(b,-x) = M(b,x), \qquad \theta(b,-x) = \tfrac{1}{2}\pi - \theta(b,x),$$

$$N(b,-x) = N(b,x), \qquad \omega(b,-x) = \tfrac{1}{2}\pi - \omega(b,x).$$

As $x \to \pm\infty$

$$M(b,x) \sim |2/x|^{1/2}, \qquad N(b,x) \sim |x/2|^{1/2}.$$

Compared with [2], §§8.6 and 8.7, the modulus functions $M(b,x)$ and $N(b,x)$ agree with their previous definitions, except when $b < 0$ and $-\sigma(b) < x < \sigma(b)$. On the other hand the phase functions agree only in the case of $\theta(b,x)$ when $b \geq 0$ and $x \geq -\sigma(b)$.

3. Approximation Theorem

The following theorem replaces both Theorems III and IV of [2]. The definitions of E, M, and N are those just given in §2.

THEOREM: *Assume that for* $a \in [0,A]$, $\zeta \in (\zeta_1,\zeta_2)$, *and each value of the positive parameter* u, *the function* $\psi(u,a,\zeta)$ *is continuous and* $\mathcal{V}_{\zeta_1,\zeta_2}(F)$ *is bounded. Then in this region the differential equation*

$$d^2w/d\zeta^2 = \{u^2(\pm a^2 - \zeta^2) + \psi(u,a,\zeta)\}w \tag{3.1}$$

has a continuous solution $w(u,a,\zeta)$, *with continuous first and second partial* ζ *derivatives, given by*

$$w(u,a,\zeta) = W(\pm\tfrac{1}{2}ua^2, \zeta\sqrt{2u}) + \epsilon(u,a,\zeta), \tag{3.2}$$

where

$$\frac{|\epsilon(u,a,\zeta)|}{M(\pm\frac{1}{2}ua^2,\zeta\sqrt{2u})} \quad , \quad \frac{|\partial\epsilon(u,a,\zeta)/\partial\zeta|}{(2u)^{1/2}N(\pm\frac{1}{2}ua^2,\zeta\sqrt{2u})}$$

$$\leq E^{-1}(\pm\tfrac{1}{2}ua^2,\zeta\sqrt{2u})\left[\exp\left\{\frac{l(\pm\frac{1}{2}ua^2)}{(2u)^{1/2}}\,\mathcal{V}_{\zeta,\zeta_2}(F)\right\} - 1\right], \tag{3.3}$$

and

$$l(b) = \sup_{x\,\epsilon(-\infty,\,\infty)} \{\Omega(x)M^2(b,x). \quad\} \tag{3.4}$$

In this result either the upper sign or the lower sign is to be taken throughout. Upper signs correspond to Theorem III of [2], and lower signs correspond to Theorem IV of [2]. In both cases a numerically satisfactory companion solution to $w(u,a,\zeta)$ may be obtained by applying the theorem to the equation obtained from (3.1) by replacing ζ by $-\zeta$. The proof of the theorem is similar to the proofs of Theorems I, II, III, and IV of [2], and it is unnecessary to record details.

The new result simplifies the general theory by replacing two theorems by a single theorem. The main advantage, however, is that error bounds are now supplied directly for all real values of ζ. In contrast, Theorem III furnishes error bounds only for nonnegative values of ζ; compare [2], §9.4. Theorem IV, however, supplies error bounds for all real values of ζ, and a property of Theorem IV that is not shared by the new theorem is that it is possible to prescribe anywhere in the closure of (ζ_1,ζ_2) a reference point ζ_0 at which $\epsilon(u,a,\zeta)$ and $\partial\epsilon(u,a,\zeta)/\partial\zeta$ both vanish. In consequence, it is conceivable that there may be applications in which Theorem IV is to be preferred. The new theorem should generally be used in place of Theorem III, however.

It should be noted that although the structure of the error bounds (3.3) is independent of the choice of ambiguous sign in (3.1), (3.2), and (3.3), the asymptotic nature of the bounds as $u \to \infty$ differs in the two cases. This is in consequence of the results

$$l(b) = O(b^{1/3}), \qquad b \to +\infty, \tag{3.5}$$

$$l(b) = O(1), \qquad b \to -\infty, \tag{3.6}$$

which correspond to similar results in §§9.2 and 10.2 of [2]. Thus, for example, if $|\psi(u,a,\zeta)/\zeta|$ is independent of u and integrable at $\zeta = -\infty$ and $+\infty$ uniformly with respect to a, then by taking the balancing function $\Omega(x)$ in the definition [2], (6.3) of the error-control function $F(u,a,\zeta)$ to be $1+|x|$, we may show that

$$\epsilon(u,a,\zeta) = E^{-1}(\tfrac{1}{2}ua^2,\zeta\sqrt{2u})\,M(\tfrac{1}{2}ua^2,\zeta\sqrt{2u})\,O(u^{-2/3}\ln u), \tag{3.7}$$

when upper signs are taken, compared with

$$\epsilon(u,a,\zeta) = E^{-1}(-\tfrac{1}{2}ua^2,\zeta\sqrt{2u})\,M(-\tfrac{1}{2}ua^2,\zeta\sqrt{2u})\,O(u^{-1}\ln u), \tag{3.8}$$

when lower signs are used. The relations (3.7) and (3.8) apply as $u \to \infty$ and are uniform with respect to $a \in [0,A]$ and $\zeta \in (-\infty,\infty)$; compare [2], §§9.3 and 10.3. Similarly for the derivative $\partial\epsilon(u,a,\zeta)/\partial\zeta$.

This research has been supported in part by the U.S. Army Research Office, Durham, Contract DA ARO D 31 124 73 G204, and the National Science Foundation, Grant GP 32841X2.

4. References

[1] Olver, F. W. J., Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders, J. Res. Nat. Bur. Stand. (U.S.), **63B** (Math. and Math. Phys.) 131–169 (1959).

[2] Olver, F. W. J., Second-order linear differential equations with two turning points, Philos. Trans. Roy. Soc. London Ser. A **278**, 137–174 (1975).

[3] Olver, F. W. J., Legendre functions with both parameters large, Philos. Trans. Roy. Soc. London Ser. A **278**, 175–185 (1975).

[4] Olver, F. W. J., Connection formulas for second-order differential equations with multiple turning points, SIAM J. Math. Anal. Vol. 8, pp. 127–154 (Feb. 1977).

(Paper 80B4–455)

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematical Sciences
Vol. 80B, No. 4, October–December 1976

A Note on Pseudointersection Graphs*

Peter J. Slater**

Institute for Basic Standards, National Bureau of Standards, Washington, D.C. 20234

(August 6, 1976)

Pseudointersection graphs are defined and a parameter called the pseudointersection number of a graph, denoted $\omega^*(G)$ and closely related to the intersection number of G, denoted $\omega(G)$, is introduced. Relations between these parameters and conditions for them to be equal are examined. The problem of computing $\omega^*(G)$ is examined.

Key words: Clique; clique graph; intersection graph; pseudointersection graph; set covering.

1. Introduction

A graph $G = (V,E)$ where V and E are the vertex and edge sets shall be considered to be a simple graph (i.e., finite, undirected and without loops or multiple edges), and all terms used shall be consistent with their definitions in [3][1].

If S is a set and $F = \{S_1,S_2, \ldots, S_p\}$ is a family of distinct nonempty subsets of S whose union is S, then the *intersection graph* of F, denoted by $\Omega(F)$, is the graph with $V\left(\Omega(F)\right) = F$ such that S_i and S_j are adjacent if and only if (iff) $i \neq j$ and $S_i \bigcap S_j \neq \emptyset$. A graph G is an intersection graph on S if there exists such a family F for which $G \approx \Omega(F)$. Every graph G is an intersection graph on some finite set [7], and the *intersection number* $\omega(G)$ is the minimum number of elements in a set S such that G is an intersection graph on S.

If $|S| = n$ then, as defined by S. Hedetniemi [5], a *representation* of G as an intersection graph on S is a one to one function, $r:V(G) \rightarrow \{0,1\}^n$, such that for $u,v \in V(G)$ one has $(u,v) \in E(G)$ iff $r(u)$ and $r(v)$ have a 1 in a common coordinate position, and if $1 \leq i \leq n$ then there is some $v \in V(G)$ such that $r(v)$ has a 1 in the ith coordinate position.

For the complete graph K_3 on vertices v_1, v_2, and v_3 we have $\omega(K_3) = 3$. If $S = \{a,b,c\}$ then one can choose, for example, $S_1 = \{a\}$, $S_2 = \{a,b\}$, and $S_3 = \{a,c\}$ or $S_1 = \{a,b\}$, $S_2 = \{b,c\}$ and $S_3 = \{a,c\}$. In the former case it is clear that elements b and c are needed only to make the S_i's distinct and do nothing to indicate adjacency. Equivalently, for $r:V(K_3) \rightarrow \{0,1\}^3$ with $r(v_1) = (1,0,0)$, $r(v_2) = (1,1,0)$ and $r(v_3) = (1,0,1)$, only the first coordinate has more than one 1 in it. As another example, the graph $K_4 - x$ is given in figure 1 as an intersection graph, and, in this case, element c of S is not necessary to indicate the adjacency of any two vertices. The size required for S can be reduced by eliminating these "fillers" used only to obtain distinct representations of each vertex.

AMS Subject Classification: 05C05.

* This work was done while the author was a National Research Council Postdoctoral Research Associate at the National Bureau of Standards, Washington, D.C. 20234. This work was supported in part by the U.S. Energy Research and Development Administration (ERDA), Contract No. AT(29-1)-789.

** Present address: Applied Mathematics Division 5121, Sandia Laboratories, Albuquerque, New Mexico 87115.

[1] Figures in brackets indicate the literature references at the end of this paper.

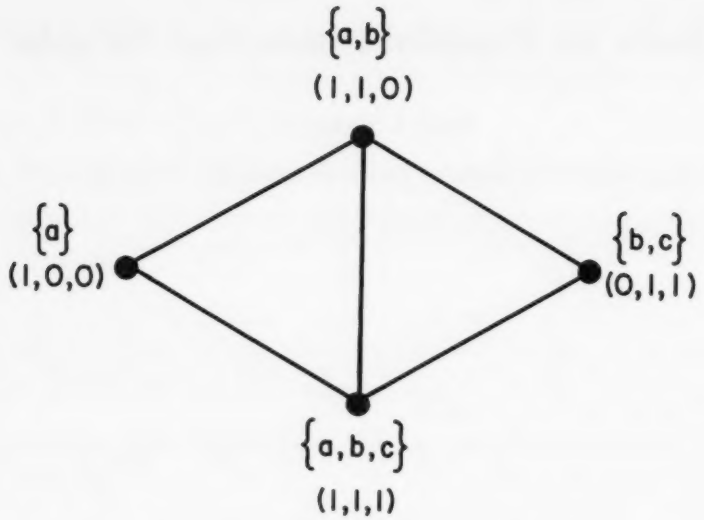

FIGURE 1. *Graph* $K_4 - x$ *as an intersection graph.*

If S is a set and $F = \{S_1, S_2, \ldots S_p\}$ is a family of subsets of S (not necessarily nonempty or distinct) whose union is S, then the *pseudointersection graph of* F, denoted by $\Omega^*(F)$, is the graph with $V\left(\Omega^*(F)\right) = F$ such that S_i and S_j are adjacent iff $i \neq j$ and $S_i \cap S_j \neq \emptyset$. Note that $S_i = \emptyset$ implies S_i corresponds to an isolated vertex. A graph G is a pseudointersection graph on S if there exists such a family F for which $G \approx \Omega^*(F)$. The *pseudointersection number of* G, denoted $\omega^*(G)$, is the minimum number of elements in a set S such that G is a pseudointersection graph on S. In particular, $\omega^*(K_t) = 0$, and $t \geq 2$ implies $\omega^*(K_t) = 1$.

If $|S| = n$ then a pseudorepresentation of G as an intersection graph on S is a function, $r:V(G) \to \{0,1\}^n$, such that for $u,v \in V(G)$ one has $(u,v) \in E(G)$ iff $r(u)$ and $r(v)$ have a 1 in a common coordinate position. The requirements that r be one to one and that $1 \leq i \leq n$ implies some $v \in V(G)$ has a 1 in the ith coordinate position have been dropped. However, if the ith component of $r(v)$ is 0 for every $v \in V(G)$, then clearly $\omega^*(G) \leq n - 1$.

2. Computing $\omega^*(G)$

Since every representation is a pseudorepresentation one obtains the following.

PROPOSITION 1: *For any graph* G, $\omega^*(G) \leq \omega(G)$.

For a graph G, $\theta(G)$ has been used to denote the minimum number of complete subgraphs of G which contain all the vertices of G. If one lets $\theta'(G)$ denote the minimum number of vertex disjoint complete subgraphs of G which contain all the vertices of G, then it is easy to see that $\theta(G) = \theta'(G)$. Now let $\theta_1(G)$ be the minimum number of complete subgraphs of G which contain all the edges of G, and let $\theta_1'(G)$ be the minimum number of edge disjoint complete subgraphs of G which contain all the edges of G. For example, $\theta(K_4 - x) = \theta_1(K_4 - x) = 2$ and $\theta_1'(K_4 - x) = 3$. Clearly $\theta_1(G) \leq \theta_1'(G)$ for every graph G. Note that $\omega^*(K_t) = 0 = \theta_1(K_t)$.

THEOREM 2: *For any graph* G, $\omega^*(G) = \theta_1(G)$.

PROOF: Suppose $\omega^*(G) = k > 0$, and let $r:V(G) \to \{0,1\}^k$ be a pseudorepresentation of G. Let S_i be the set of all vertices v in $V(G)$ for which $r(v)$ has a 1 in the ith coordinate $(1 \leq i \leq k)$. Now the subgraph generated by S_i, denoted $\langle S_i \rangle$, is complete since $u,v \in S_i$ implies u and v have a 1 in a common coordinate position. If $(u,v) \in E(G)$ then u and v have a 1 in some common coordinate, say the ith. Hence $(u,v) \in \langle S_i \rangle$. Thus $\langle S_1 \rangle, \langle S_2 \rangle, \ldots, \langle S_k \rangle$ are complete subgraphs containing every edge of G, and $\theta_1(G) \leq \omega^*(G)$.

Suppose $\theta_1(G) = k$, and let $S_1, S_2, \ldots, S_k$ be the point sets of complete subgraphs such that every edge of G is in some $\langle S_i \rangle$. Define $r: V(G) \to \{0,1\}^k$ by $r(v) = (e_1, e_2, \ldots, e_k)$ where $e_i = 1$ if $v \in S_i$ and $e_i = 0$ if $v \in S_i$. It is easy to see that r is a pseudorepresentation of G. Thus $\omega^*(G) \leq \theta_1(G)$.

While $\theta_1 = \omega^*$, θ_1' and ω are independent parameters. For example, $\omega^*(K_t) = \theta_1(K_t) = \theta_1'(K_t) = 1 < \omega(K_t)$ when $t \geq 2$. For the graph G of figure 2, $\theta_1(G) = \omega^*(G) = \omega(G) = 3$ and $\theta_1'(G) = 4$.

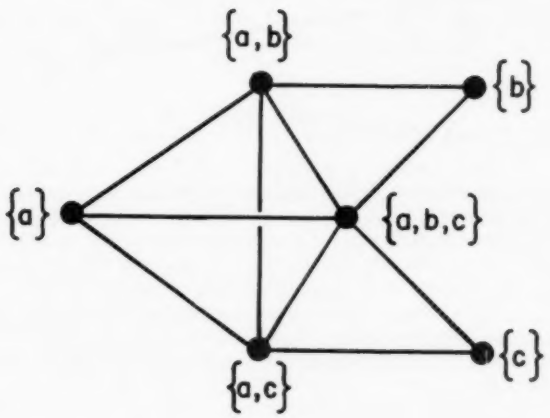

FIGURE 2. *A graph with* $\omega(G) < \theta_1'(G)$.

If $v \in V(G)$ then the *neighborhood of* v, denoted $N(v)$, is the set of vertices adjacent to v, and the *closed neighborhood of* v, denoted $N[v]$, is $N(v) \cup \{v\}$.

THEOREM 3: *If* G *has no isolated vertices and for any two distinct vertices,* u *and* v, $N[u] \neq N[v]$, *then* $\omega^*(G) = \omega(G)$.

PROOF: Let $r: V(G) \to \{0,1\}^n$ be a pseudorepresentation of G where $\omega^*(G) = n$. Suppose $r(v_1) = r(v_2)$. Since G has no isolated vertices, $r(v_1) \neq (0,0,\ldots,0)$. Hence $r(v_1)$ and $r(v_2)$ have a 1 in a common coordinate and so v_1 and v_2 are adjacent. Now u is adjacent to v_1 iff $r(u)$ and $r(v_1) = r(v_2)$ have a 1 in a common coordinate iff u is adjacent to v_2. Thus $N[v_1] = N[v_2]$.

This contradiction implies that r is a one to one function. As already noted, there cannot be an i with $1 \leq i \leq n$ such that $r(v)$ is 0 in the ith component for every $v \in V(G)$. That is, r is actually a representation. This implies $\omega(G) \leq \omega^*(G)$. Consequently $\omega(G) = \omega^*(G)$.

COROLLARY 3.1.1: *If* G *is triangleless and each component has at least three vertices, then* $\omega(G) = \omega^*(G)$.

The following is an easy consequence of the fact that $\omega^*(G) = \theta_1(G) \leq q$ where q is the number of edges of $G (q = |E(G)|)$.

PROPOSITION 4: *Graph* G *is triangleless iff* $\omega^*(G) = q$.

The graph G in figure 3 gives a counterexample to the converse of Theorem 3 since $\omega(G) = \omega^*(G) = 3$ and, while G has no isolated vertices, the vertices $S_1 = \{a,b,c\}$ and $S_2 = \{a,b\}$ satisfy $N[S_1] = N[S_2]$. Consideration of edges x, y, and z show that $\theta_1(G) \geq 3$. In general, if $\epsilon(G)$ is the maximum number of edges, no two of which are in a common clique, then clearly $\theta_1(G) \geq \epsilon(G)$. (A *clique* is a maximal complete subgraph.)

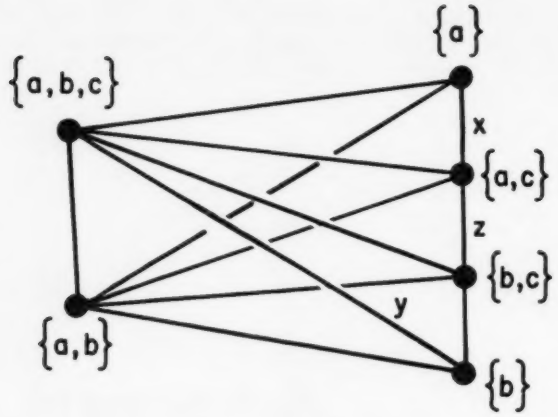

FIGURE 3. *A graph with* N[{a,b,c}] = N[{a,b}] *and* $\omega(G) = \omega^*(G)$.

To evaluate $\theta_1(G)$, consider the set of complete subgraphs selected to cover $E(G)$. Since every complete subgraph is contained in a clique, $\theta_1(G)$ can be defined as the minimum number of cliques of G which contain all the edges of G. Let $C_1, C_2, \ldots, C_t$ be the cliques of G, and let V_i and E_i be the vertex and edge sets, respectively, of C_i. The *clique* graph of G, denoted $C(G)$, is the intersection graph on $V(G)$ with $F_1 = \{V_1, V_2, \ldots, V_t\}$; let the *clique-edge graph*, denoted $C(G)$, be the intersection graph on $E(G)$ with $F_2 = \{E_1, E_2, \ldots, E_t\}$. Thus $C(G)$ can be considered to be the graph whose vertices are the cliques of G, with two cliques adjacent iff they have an edge in common. If G has no isolated vertices, then it can be seen that $C(G)$ is obtained from $C(G)$ by deleting each edge corresponding to two cliques intersecting in exactly one point.

The work of Hamelink [4] and Roberts and Spencer [8] gives us necessary and sufficient conditions for a graph H to be the clique graph of some graph G. These same conditions can be shown to be necessary and sufficient for H to be the clique-edge graph of some graph F. In general, let $C_k(G)$ be the graph whose vertices are the cliques of G, with two cliques adjacent iff they have at least k vertices in common. Given H then there is a graph F with $H = C_k(F)$ iff there is a graph G with $H = C(G)$ (See [8].)

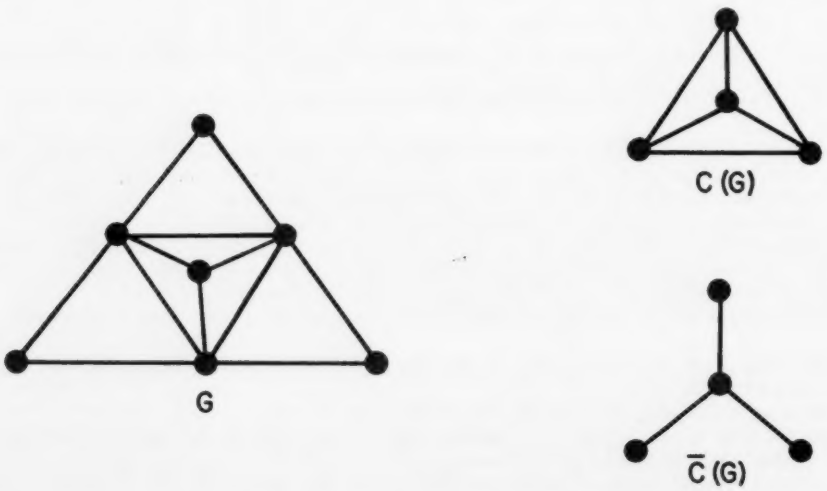

FIGURE 4. *A graph G and its clique graphs.*

In finding $\theta_1(G)$ one has a collection of subsets of $E(G)$, namely $\{E_1,E_2,\ldots,E_t\}$, and one needs to select a subcollection with the smallest number of elements that still covers $E(G)$. Thus evaluating the pseudointersection number of G is a set covering problem. Much work has been done on set covering problems using integer programming, for example, by Garfinkel and Nemhauser [2, chapter 8]. The "reductions" possible for set covering problems lead to bounds for $\theta_1(G)$.

For example, consider the following. Let p_i be the vertex of $C(G)$ corresponding to E_i for $1 \leq i \leq t$. Suppose that there is an edge e_j in E_j such that $e_j \in E_i$ when $j \neq i (1 \leq i \leq t)$ iff $1 \leq i \leq k$. Let $a_0(G)$ denote the minimum number of vertices of graph G such that every edge of G is incident with at least one of these vertices.

PROPOSITION 5: $k \leq \theta_1(G) = \omega^*(G) \leq k + a_0\left(C(G) - \{p_1,\ldots,p_k\}\right)$.

PROOF: Suppose $E_i (1 \leq i \leq t)$ and $e_j (1 \leq j \leq k)$ are as described above. Clearly E_1, $E_2,\ldots,E_k$ must be chosen to cover $e_1,e_2,\ldots,e_k$, and so $k \leq \theta_1(G)$. Now for any edge e of G in two or more cliques there are two or more adjacent points of $C(G)$, say $u_1,u_2,\ldots$. If one of these is a $p_j (1 \leq j \leq k)$, then e is in the clique E_j already selected. If not, then edge (u_1,u_2) is in $C(G) - \{p_1,\ldots,p_k\}$. Consequently $C_1,\ldots,C_k$ and the cliques corresponding to an a_0 set of $C(G) - \{p_1,\ldots,p_k\}$ cover all the edges of G.

3. Observations

In addition to the advantages of pseudorepresentations over representations obtained by a direct concentration on adjacency requirements, there are also situations in which pseudointersection graphs can be formed while intersection graphs cannot. Given any m by n 0, 1-matrix M, one may, for example, form a pseudointersection graph on m vertices $v_1,v_2,\ldots,v_m$ by using a pseudorepresentation $r:\{v_1,\ldots,v_m\} \rightarrow \{0,1\}^n$ where $r(v_i)$ is the ith row of M. Very often r will not be a representation. Likewise one can reverse the rows and columns (that is, treat M^T as above). As an example, one obtains the line graph of G if the transpose of the incidence matrix is used.

Often one forms a graph H from a given graph G by a fixed procedure, such as forming the line graph or clique graph. Much recent work has been done to investigate what happens when the operation is iterated, for example in [1] and [6]. Such iterations on the formation of pseudointersection graphs will be examined in [9].

4. References

[1.] Dongre, N. M., On the squares of cycles and other graphs, Indian Journal of Pure and Appl. Math. **4,** 1–4 (1973).
[2.] Garfinkel, R. S., and Nemhauser, G. L., Integer Programming, (John Wiley and Sons, New York, 1972).
[3.] Harary, F., Graph Theory (Addison-Wesley, Reading, Mass., 1969).
[4.] Hamelink, R. C., A partial characterization of clique graphs, Journal of Combinatorial Theory **5,** 192–197 (1968).
[5.] Hedetniemi, S. T. personal communication.
[6.] Hedetniemi, S. T., and Slater, P.J., Line graphs of triangleless graphs and iterated clique graphs, in Graph Theory and Applications (Springer-Verlag, Berlin, 1972), pp. 139–149.
[7.] Marczewski, E., Sur deux proprietes des classes d'ensembles, Fund. Math. **33,** 303–307 (1945).
[8.] Roberts, F. S., and Spencer, J. H., A characterization of clique graphs, Journal of Combinatorial Theory **10,** 102–108 (1971).
[9.] Slater, P. J., Pseudointersection operators, in preparation.

(Paper 80B4–456)

JOURNAL OF RESEARCH of the National Bureau of Standards—A. Mathematical Sciences
Vol. 80B, No. 4, October–December 1976

An Efficient Linear Algebraic Algorithm for the Determination of Isomorphism in Pairs of Undirected Graphs

Charles R. Johnson* and Frank Thomson Leighton

Institute for Basic Standards, National Bureau of Standards, Washington, D.C. 20234

(September 29, 1976)

An algorithm, complete with a specific FORTRAN implementation, is presented for the problem of determining whether or not two undirected graphs are isomorphic. The algorithm, centered upon the eigenvalues and eigenvectors of a modified adjacency matrix and techniques for decreasing the size of the automorphism group, is quite different from others (most of which are combinatorially based) and tends to work relatively very quickly on difficult test cases as well as on typical examples. Complexity estimates are given for many eventualities.

Key words: Graph isomorphism; labels; modified adjacency matrix; spectrum.

1. Introduction

Two undirected graphs are said to be *isomorphic* if there exists a one-to-one correspondence between their nodes which preserves adjacency. A computationally difficult issue in the theory of graphs is the determination of isomorphism. For example the graphs in figure 1a are pairwise isomorphic while those in 1b are not.

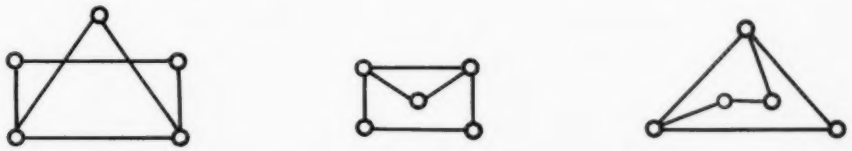

Figure 1a

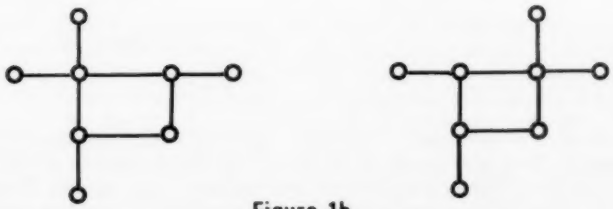

Figure 1b

AMS Subject Classification: 68A10 05C99, 65F99.

* Present address: Institute for Physical Science and Technology, University of Maryland, College Park, Md. 20742.

The ability to divide a set of graphs into maximal subsets whose elements are pairwise isomorphic has several immediate applications. If nodes are identified with atoms of a particular compound and if edges are identified with the existence of a chemical bond, then, for instance, such an ability is of use to the chemist in distinguishing among many theoretically produced compounds. Other applications include physics (e.g. "hearing" the shape of a drum [1])[1], electronic circuit theory, linguistics etc.

Much effort has been devoted to the development of an algorithm which will determine whether or not any two graphs are isomorphic in an amount of time that is bounded by a finite polynomial in the number of nodes of the graphs considered. At the moment, the authors are unaware of the existence of any such algorithm. Many approximate or heuristic procedures have been developed however. Many such algorithms check conditions combinatorially necessary for isomorphism, some also attempt to construct a permutation that would exhibit isomorphism. Corneil presents a review of many such techniques. All known exact methods grow exponentially in required time. Unfortunately, as pointed out in [4] such procedures could take very long to determine whether or not two graphs of order 15 are isomorphic. Hence, such methods are clearly inadequate for large order graphs.

If a computer is to be used to determine whether or not two graphs are isomorphic, the graphs must be represented in a form suitable for machine processing. The adjacency matrix of a graph is such a standard form. First, the nodes of the graph are arbitrarily associated with the integers 1,2,3 . . . n where n is the order of the graph. The (i,j) component of the adjacency matrix is then defined to be 1 if there is an edge connecting the ith node and the jth node and is 0 otherwise. The (i,i) entry commonly contains the label given the node. In the case of an unlabeled graph, (i,i) entries are typically zero. While different orderings of the nodes produce different adjacency matrices, two adjacency matrices are isomorphic if and only if, independent of labeling, they correspond to the same graph. Figure 2 illustrates this.

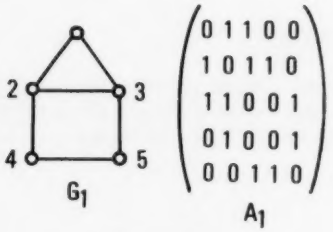

Figure 2a

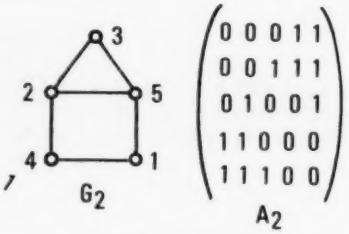

Figure 2b

$$A_2 = P^T A_1 P \text{ Where } P = \begin{pmatrix} 0&0&1&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&1 \\ 0&0&0&1&0 \\ 1&0&0&0&0 \end{pmatrix}$$

Figure 2c

The algorithm about to be described is divided roughly into three sections each satisfying a specific purpose. The first section consists of several basic tests for nonisomorphism. These tests are all $0(n^3)$ or better and are surprisingly effective in distinguishing nonisomorphic graphs. Since nearly all pairs of graphs that pass through the initial tests undistinguished are likely to be isomorphic, the second part of the algorithm involves the attempted construction of a permutation between the adjacency matrices of the graphs. In the great majority of cases, this is easily and quickly accomplished. The second section simultaneously serves as a more complicated test for nonisomorphism should it be found impossible to generate a satisfactory permutation.

[1] Figures in brackets indicate the literature references at the end of this paper.

The final section is specifically designed to deal with certain classes of matrices that do not easily yield to the tests or efforts to construct a permutation found in the previous two components. In general the tests in this section are more time consuming, generally $0(n^4)$, and are quite detailed. Methods of finding a satisfactory permutation are included as well as tests for nonisomorphism.

A complete FORTRAN listing of the algorithm, including explanatory comments, follows its description in section 3 and the test results provided in section 4 demonstrate the efficiency of the algorithm.

2. The Algorithm

Initially, the algorithm employs four basic tests, each of which checks for the failure of a condition necessary for isomorphism. The first test determines whether or not the node labels of the two graphs are identical. If they are not, then the corresponding graphs are not isomorphic. In the case of unlabeled graphs, the test yields no information and is unnecessary. This test is performed by SUBROUTINE LABEL and takes $0(n^2)$ amount of time when n is the order of the graphs.

A more useful test involves the comparison of the node valences of the two graphs. The valence of a node is the number of edges connected to it. The valence of each node, once computed, is combined with the node's label to form a new label for that node. Two nodes of a graph will have the same new label if and only if they have the same valence as well as identical initial labels. Thus two graphs with different sets of new labels are themselves nonisomorphic. For example, the graphs in figure 3 share the same set of node valences and of initial labels yet when these two qualities are considered mutually, the graphs are found to be nonisomorphic.

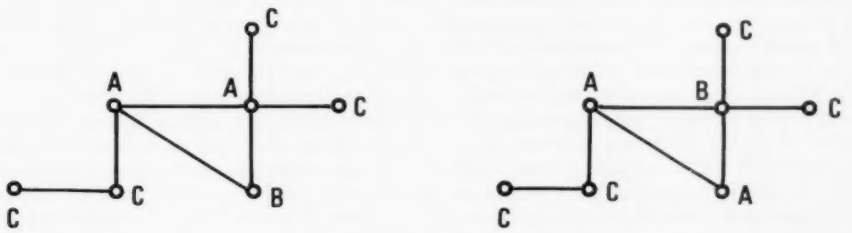

Figure 3

The set of initial node labels is (A,A,B,C,C,C,C) for each graph. Similarly the set of node valences is (1,1,1,2,2,3,4) in each case. When combined to produce sets of new labels, however, the following two distinct sets are formed: (A3,A4,B2,C1,C1,C1,C2) and (A2,A3,B4,C1,C1,C1,C2). Since these two sets are clearly not identical, the two graphs are not isomorphic. This test takes $0(n^2)$ time and is performed in SUBROUTINE VALENC.

The third basic test involves the search for and labeling of duplicate nodes. A given node is called an ordinary duplicate of degree k if there exist $k - 1$ and no more than $k - 1$ other identically labeled nodes in the graph which are connected by edges to the same nodes as is the given node. A given node is called a connected duplicate of degree k if the $k - 1$ nodes just mentioned are all mutually connected. To further illustrate consider the graph in figure 4. The numbers are only for use in referencing the nodes while the letters correspond to actual labels.

Nodes 1 and 3 are connected duplicates of degree 2. Nodes 2 and 4 are ordinary duplicates of degree 2. Nodes 6 and 7 are not duplicates since they have different labels. Finally nodes 10, 11, and 12 are ordinary duplicates of degree 3. All other nodes are trivially duplicates of degree 1.

Node duplicate degrees are easily found from the 0,1 adjacency matrix of a graph. Groups of ordinary duplicate nodes correspond to groups of identical rows (with identical labels) of the adjacency matrix with 0's entered on the diagonal. If 1's are entered on the diagonal, then the connected duplicates can be similarly found. It is easily seen that two graphs are not isomorphic if the ordinary and connected duplicate degrees of their nodes differ in any manner.

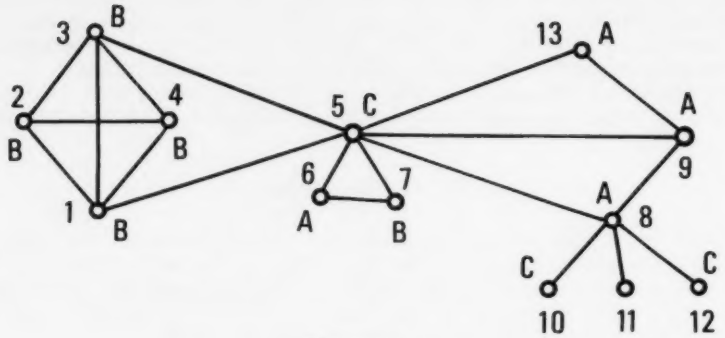

Figure 4

As before, the node labels of each graph are updated to reflect the newly obtained information and then compared. This is done in SUBROUTINE DUPLIC and takes $0(n^3)$ time.

While the identification of duplicate nodes serves as a worthwhile test for nonisomorphism, a potentially much more useful purpose is also served. A graph with a large number of nodes with duplicate degrees greater than one yields an adjacency matrix, A, and a large number of nontrivial automorphisms, that is permutations P such that $P^TAP = A$. For reasons explained further on, graphs whose adjacency matrices have this property are especially difficult to analyze and any procedure which efficiently reduces the number of such permutations through appropriate labeling of the nodes of the graph, will greatly reduce the complexity of following computations. Labeling of duplicate nodes was found to be a particularly fast and simple way to achieve this end for many graphs. It should be noted that this labeling alone does not necessarily reduce the automorphism group to the identity.

A well-known necessary condition for isomorphism involves the comparison of spectra (set of eigenvalues) of the 0,1 adjacency matrices of two graphs. Much effort has been devoted to nonisomorphic pairs for which the 0,1 adjacency matrices have the same spectrum. Such pairs of graphs are called cospectral [5]. Many cospectral pairs of graphs are distinguishable, however, through comparison of the spectra generated from adjacency matrices with numbers different from 0,1 used to represent nonedges and edges [6]. It appears that the use of 0 and 1 in the computation of the spectra of graphs ignores much of their inherent structure. A greater amount of information about the graph can be discovered from the examination of the spectra of adjacency matrices with 1's representing no edges, and a variable x representing an edge. Due to the nature of the characteristic polynomial, no generality is lost by replacing x with either a transcendental number or certain large functions of the number of nodes in the graph. Unfortunately neither option is feasible for use with the computer since a transcendental number is truncated to a rational and a large value of x yields errors in computation small in comparison to x but large in comparison to 1. A compromise solution was developed that allows x to be a function that increases linearly with the order of the graph yet is essentially as effective in distinguishing nonisomorphic graphs as is the more general procedure.

In order to utilize the information stored in the updated labels of the nodes, the diagonal elements of the adjacency matrices are assigned the value of the appropriate label. The eigenvalues of each modified adjacency matrix are then computed and compared. Should the resulting spectra not be identical, the graphs are not isomorphic. The preparation of the adjacency matrices for eigenvalue computation is effected in SUBROUTINE PREPAR and takes $0(n^2)$ time. The computation of the eigenvalues as well as the eigenvectors (for future use) of each matrix is carried out in SUBROUTINE SPECTR and takes $0(n^3)$ time.

Since two graphs that are not distinguished by the preceding sequence of tests may well be isomorphic, it is reasonable at this point to attempt to generate a permutation that relates the two graphs. The number of such possible permutations can be greatly limited with efficient use of the information stored in the eigenvectors of the two adjacency matrices. If A and B are two symmetric matrices with identical spectra then there exist orthogonal matrices U and V and a diagonal matrix D such that

$$U^TAU = D \quad \text{and} \quad V^TBV = D.$$

Further assuming that A and B are permutationally equivalent we have $B = P^TAP$ where P is a permutation matrix. Then:

$$U^TAU = V^TP^TAPV$$

$$D = V^TP^TUDU^TPV$$

$$U^TPVD = DU^TPV$$

and thus D commutes with $E = U^TPV$.

If $E = (e_{ij})$ and $D = (d_{ij})$ with $d_{ij} = 0$ for $i \neq j$, then $DE = (d_{ii}e_{ij})$ and $ED = (d_{jj}e_{ij})$. Thus if $d_{ii} \neq d_{jj}$, then $d_{jj}e_{ij} = d_{ii}e_{ij} \rightarrow e_{ij} = 0$.

Thus $E =$ $\begin{pmatrix} E_1 & & & 0 \\ & E_2 & & \\ & & \cdot & \\ 0 & & & \cdot \\ & & & & E_m \end{pmatrix}$

where E_i is an orthogonal matrix of degree equal to the multiplicity of the ith distinct eigenvalue of A.

The relation $UE = PV$ serves to greatly restrict the number of permutations, P. Consider first the restrictions imposed by the existence of eigenvalues of multiplicity one. In this case, the corresponding matrix E_i is trivially (1) or (−1). Thus for each j such that d_{jj} is unique, we know that the set of permutations of

$$\begin{pmatrix} u_{1j} \\ u_{2j} \\ \cdot \\ \cdot \\ \cdot \\ u_{nj} \end{pmatrix} \text{ into } \begin{pmatrix} v_{1j} \\ v_{2j} \\ \cdot \\ \cdot \\ \cdot \\ v_{nj} \end{pmatrix} \text{ combined with}$$

$$\text{those of } \begin{pmatrix} -u_{1j} \\ -u_{2j} \\ \cdot \\ \cdot \\ \cdot \\ -u_{nj} \end{pmatrix} \text{ into } \begin{pmatrix} v_{1j} \\ v_{2j} \\ \cdot \\ \cdot \\ \cdot \\ v_{nj} \end{pmatrix} \text{ include}$$

all permutations of A into B.

$$\begin{pmatrix} u_{1j} \\ u_{2j} \\ \cdot \\ \cdot \\ \cdot \\ u_{nj} \end{pmatrix} \text{ and } \begin{pmatrix} v_{1j} \\ v_{2j} \\ \cdot \\ \cdot \\ \cdot \\ v_{nj} \end{pmatrix} \text{ are simply the}$$

eigenvectors of A and B corresponding to the eigenvalue d_{jj} and are easily computed.

Since the components of the eigenvectors of most modified adjacency matrices are unique, the number of possible permutations of A into B is small. In these cases, all possible permutations are quickly generated and checked until one is found that relates A and B or until the supply is exhausted. In the latter case, the graphs are not isomorphic.

As was mentioned earlier graphs that yield an adjacency matrix, A, and several permutations, P, such that $P^TAP = A$ are very difficult to work with. The reason for this phenomenon now becomes apparent. If the number of permutations of A into itself is large, then there will be many pairs of k and l such that $u_{kj} = u_{lj}$ for every j such that d_{jj} is unique. This, in turn, greatly increases the number of permutations

$$\text{of} \quad \begin{pmatrix} u_{1j} \\ u_{2j} \\ \cdot \\ \cdot \\ \cdot \\ u_{nj} \end{pmatrix} \quad \text{into} \quad \begin{pmatrix} v_{1j} \\ v_{2j} \\ \cdot \\ \cdot \\ \cdot \\ v_{nj} \end{pmatrix} \quad \text{and thus}$$

the amount of time which may be required to find a permutation of A into B and certainly the amount of time required to exhaust all possibilities is greatly increased. As an example, certain stochastic adjacency matrices have only one eigenvalue of multiplicity one. In many of these cases, the corresponding eigenvector components are all identical. In this, as well as in other less severe cases, the above described method of finding a permutation of A into B is not practical.

For such graphs a more sophisticated, as well as time consuming, approach is required. Again referring to the relation $UE = PV$, it is useful to examine the case when an eigenvalue has multiplicity two. In other words, assume that $d_{jj} = d_{kk}$ for $k = j, j+1$ but $d_{jj} \neq d_{kk}$ for all other k. Then we know that

$$\begin{pmatrix} u_{1j} & u_{1j+1} \\ u_{2j} & u_{2j+1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ u_{nj} & u_{nj+1} \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = P \begin{pmatrix} v_{1j} & v_{1j+1} \\ v_{2j} & v_{2j+1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ v_{nj} & v_{nj+1} \end{pmatrix}$$

or that

$$\begin{pmatrix} u_{1j} & u_{1j+1} \\ u_{2j} & u_{2j+1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ u_{nj} & u_{nj+1} \end{pmatrix} \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = P \begin{pmatrix} v_{1j} & v_{1j+1} \\ v_{2j} & v_{2j+1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ v_{nj} & v_{nj+1} \end{pmatrix}$$

In each case, there are n possible values for a. Selecting i such that either $u_{ij} \neq 0$ or $u_{ij+1} \neq 0$ and assuming that there is a permutation of A into B which takes the ith node of A into the kth node of B, we have

$$\begin{aligned} u_{ij}a + u_{ij+1}b &= v_{kj} \\ -u_{ij}b + u_{ij+1}a &= v_{kj+1} \end{aligned}$$

or

$$\begin{aligned} u_{ij}a + u_{ij+1}b &= v_{kj} \\ u_{ij}b - u_{ij+1}a &= v_{kj+1} \end{aligned}$$

which yields

$$\begin{pmatrix} u_{ij} & u_{ij+1} \\ u_{ij+1} & -u_{ij} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} v_{kj} \\ v_{kj+1} \end{pmatrix}$$

or

$$\begin{pmatrix} u_{ij} & u_{ij+1} \\ -u_{ij+1} & u_{ij} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} v_{kj} \\ v_{kj+1} \end{pmatrix}.$$

Solving for a and b yields:

$$a = \frac{u_{ij}\, v_{kj} + u_{ij+1}\, v_{kj+1}}{u_{ij}^2 + u_{ij+1}^2},$$

$$b = \frac{-u_{ij}\, v_{kj+1} + u_{ij+1}\, v_{kj}}{u_{ij}^2 + u_{ij+1}^2}$$

or

$$a = \frac{u_{ij}\, v_{kj} - u_{ij+1}\, v_{kj+1}}{u_{ij}^2 + u_{ij+1}^2} ,$$

$$b = \frac{u_{ij}\, v_{kj+1} + u_{ij+1}\, v_{kj}}{u_{ij}^2 + u_{ij+1}^2} .$$

Note that $a^2 + b^2 = 1$ so that $|a| \leq 1$. Thus the permutations P are limited to those that satisfy

$$a \begin{bmatrix} u_{1j} \\ u_{2j} \\ \cdot \\ \cdot \\ \cdot \\ u_{nj} \end{bmatrix} + b \begin{bmatrix} u_{1j+1} \\ u_{2j+1} \\ \cdot \\ \cdot \\ \cdot \\ u_{nj+1} \end{bmatrix} = P \begin{bmatrix} v_{1j} \\ v_{2j} \\ \cdot \\ \cdot \\ \cdot \\ v_{nj} \end{bmatrix}$$

for any of the $2n$ pairs of (a,b), two pairs corresponding to each value of k. For each (a,b), the problem then reduces to the previously examined case of eigenvalues of multiplicity one. The only difference is that the process must be repeated $2n$ times before all possible permutations have been checked.

SUBROUTINE OPTEIG selects an optimal eigenvector, one that has as few multiple components as possible, and takes $0(n^3)$ time. If the selected eigenvector corresponds to an eigenvalue of multiplicity one, SUBROUTINE SINGLE is called upon. This subroutine is $0(n^3)$ but the leading coefficient can be very large if there are large numbers of multiple eigenvector components. SUBROUTINE DOUBLE calls SUBROUTINE SINGLE at most $2n$ times and thus is $0(n^4)$ with the possibility of a very large leading coefficient.

Due to the complex nature of the relation $UE = PV$ for blocks of E of dimension greater than two, little progress was made in obtaining meaningful data from the eigenvectors corresponding to eigenvalues of multiplicity greater than two.

Should neither the eigenvectors corresponding to eigenvalues of multiplicity one nor those of multiplicity two sufficiently restrict the number of permutations possible, still other algebraic approaches can be employed.

One such approach is the examination of the spectra of the n subgraphs of order $n - 1$ of the original graph. An obvious necessary condition for isomorphism is that the two graphs generate the same sets of spectra. Further there is the possibility that in a given set of spectra, many of them will be unique, thus creating the possibility of generating and testing all possible permutations in a small amount of time. Distinguishing node duplicates again helps to reduce the maximum number of permutations possible. Experience with a limited number of large stochastic matrices has shown, however, that this procedure is far more effective as a nonisomorphism test than as one that attempts to generate and test all possible permutations. This procedure is contained in SUBROUTINE SUBVAL and takes $0(n^4)$ time.

Carrying the eigenvector analysis further all possible permutations can be derived through examination of eigenvectors of subgraphs of the original graph. This method has proved to be one of the most successful in distinguishing very difficult large, stochastic graphs. For simplicity and speed, all submatrices of order $n - 1$ are searched for the optimum or a sufficiently satisfactory eigenvector corresponding to an eigenvalue of multiplicity one. This eigenvector is then compared with the corresponding eigenvector generated from each of the subgraphs of the second graph that has a spectrum matching the spectrum of the subgraph which generated the optimal eigenvector. This procedure is executed in SUBROUTINE SUBVEC. The order is $0(n^4)$ as SUBROUTINE SINGLE can be called a maximum of n times. This approach tends to be more time consuming than the others mentioned since it could require the generation of n eigenvector matrices.

In a similar fashion, one could examine the eigenvectors of subgraphs corresponding to eigenvalues of multiplicity two. Another possible approach would be to examine the n^2 spectra generated by sequentially generating all possible subgraphs of order $n - 2$. These and other possible procedures are $0(n^5)$ or worse and very time consuming. It does not appear that the trade off of speed for effectiveness dictates inclusion of such unwieldly approaches in the algorithm.

3. Computer Implementation

The algorithm was organized into fourteen subroutines and coded in FORTRAN V for use with the UNIVAC 1108 computer. SUBROUTINE GISOM serves as the coordinating subroutine and is the

only subroutine that need be referenced by a main program. Each subroutine listing is fully commented, indicating values of variables on input, output and during the life of the subroutine.

All eigenvalues and eigenvectors were calculated using the EISPACK software described in [9]. Reference to [9] should be made for a detailed explanation of the function, running time, error messages and accuracy of the EISPACK subroutines. Throughout the program two numbers, A and B are considered identical if $||A| - |B|| <$ MACHP * $(1 + |A|)$. The precision constant, MACHP, is determined and set by the user in the call to GISOM. For use with the EISPACK software on the UNIVAC 1108, a value of MACHP $= 0.0003$ was found to be satisfactory.

Another user option in the call to GISOM command sets a limit on the number of permutations that may be tested for a given pair of graphs. Generally, this value, MAXT, can be set to a very large number, say 5000, since very little time is expended on each test. MAXT is of greatest use when it is desired to sequentially test several pairs of very large matrices without spending too much time on any one pair.

Time rather than storage conservation was emphasized. The subroutines utilize five $n \times n$ arrays and several single dimension $0(n)$ arrays, all of which are passed as part of the subroutine call statements. There are no common blocks and no input-output statements in the subroutine as all pertinent data is returned through the CALL GISOM command in the main program. The STATUS and ERR variables serve as the key to what happened when the two graphs were tested for isomorphism. The other variables contain a variety of data whose meaning depends on the value of ERR and STATUS. This data can often be quite useful in determining exactly why two graphs are not isomorphic. The interpretation of ERR and STATUS is given with the commentary of each subroutine. A list of the program follows.

```
      SUBROUTINE GISOM (A,B,C,XA,XB,DA,DB,EA,EB,FA,FB,SC,SD,SE,CE,CF,G,H
     +                  ,P,STATUS,ERR,MACHP,MAXT,N,NM,NT)
C
C  THIS SUBROUTINE IS THE CONTROL SUBROUTINE FOR THE ALGORITHM.
C
C  ON INPUT:
C     A AND B CONTAIN THE 0,1 FORM OF THE ADJACENCY MATRICES OF THE TWO
C        GRAPHS TO BE COMPARED.  THE DIAGONAL ELEMENTS CONTAIN LABELS
C        WHICH MAY RANGE IN VALUE FROM 0 TO 99.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.  A
C        VALUE OF .0003 IS SUGGESTED WHEN THE PROGRAM IS RUN ON A
C        MACHINE COMPARABLE TO THE UNIVAC 1108 AND WITH SOFTWARE
C        EQUIVALENT TO EISPACK IN ACCURACY.
C     MAXT IS APPROXIMATELY THE MAXIMUM NUMBER OF PERMUTATIONS THAT MAY
C        BE TESTED.  EXCEPT IN CASES OF SEVERE TIME RESTRICTIONS, A
C        VALUE OF 5000 IS SUGGESTED.
C     N IS THE ORDER OF THE GRAPHS TO BE COMPARED.
C     NM IS THE DIMENSION OF MOST OF THE ARRAYS.
C     NT = 2*NM AND IS THE DIMENSION OF CE AND CF.
C
C  ON RETURN:
C     A AND B ARE POSSIBLY DESTROYED.
C     P CONTAINS A PERMUTATION SUCH THAT A(I,J)=B(P(I),P(J)) FOR ALL I,J
C        SHOULD SUCH A PERMUTATION BE FOUND TO EXIST.
C     STATUS RANGES IN VALUE FROM 1 TO 10 AND RECORDS THE POINT IN GISOM
C        WHERE THE ALGORITHM TERMINATED.  FOR EXAMPLE, A VALUE OF STATUS
C        =3 MEANS THAT THE ALGORITHM TERMINATED FOLLOWING THE NODE
C        VALENCE CHECK WHICH IMPLIES THAT THE NODE VALENCES OF THE TWO
C        GRAPHS DIFFERED AND THUS THAT THE TWO GRAPHS ARE NOT
C        ISOMORPHIC.
C     ERR CONTAINS INFORMATION PERTAINING TO THE REASON FOR THE
C        TERMINATION OF THE ALGORITHM.  AN EXPLANATION OF ITS MEANING
C        CAN BE FOUND WITH THE COMMENTS IN THE SUBROUTINE THAT WAS LAST
C        CALLED BY GISOM.  FOR EXAMPLE, A STATUS VALUE OF 7 COMBINED
C        WITH AN ERR OF 0 IMPLIES THAT A PERMUTATION WAS FOUND THAT
C        RELATES A AND B THROUGH USE OF AN EIGENVECTOR CORRESPONDING TO
C        AN EIGENVALUE OF MULTIPLICITY ONE.
C     MACHP, MAXT, N, NM, AND NT ARE UNCHANGED.
C
C  ALL OTHER VARIABLES ARE CLASSIFIED AS WORKING VARIABLES.  SUCH
C  VARIABLES MAY CONTAIN RELEVANT INFORMATION DEPENDING ON THE VALUES OF
C  STATUS AND ERR.
C
      INTEGER N,NM,NT,ERR,MAXT,G(NM),H(NM),P(NM),STATUS,JA
      REAL A(NM,NM),B(NM,NM),C(NM,NM),XA(NM,NM),XB(NM,NM),DA(NM),DB(NM),
     +     EA(NM),EB(NM),FA(NM),FB(NM),SC(NM),SD(NM),SE(NM),CE(NT),MACHP
     +     ,CF(NT)
C
      ERR=0
      STATUS=1
      IF (N.GT.NM) RETURN
C
C  COMPARE THE NODE LABELS OF THE TWO GRAPHS.
C
      STATUS=2
      CALL LABEL (A,B,DA,DB,SC,ERR,N,NM)
      IF (ERR.GT.0) RETURN
C
C  COMPARE THE NODE VALENCES AND UPDATE THE CORRESPONDING NODE LABELS.
```

```
C
      STATUS=3
      CALL VALENC (A,B,DA,DB,SC,ERR,N,NM)
      IF (ERR.GT.0) RETURN
C
C  FIND, LABEL, AND COMPARE ORDINARY AND CONNECTED DUPLICATE NODES.
C
  10  STATUS=STATUS+1
         CALL DUPLIC (A,B,DA,DB,SC,SD,ERR,STATUS,N,NM)
         IF (ERR.GT.0) RETURN
      IF (STATUS.EQ.4) GO TO 10
C
C  CONVERT THE 0,1 FORM OF THE ADJACENCY MATRIX TO THE 1,X FORM.
C
      CALL PREPAR (A,B,DA,DB,N,NM)
C
C  COMPUTE AND COMPARE THE SPECTRA OF THE TWO ADJACENCY MATRICES.
C
      STATUS=6
      CALL SPECTR (A,B,XA,XB,EA,EB,SC,G,ERR,MACHP,N,NM)
      IF (ERR.NE.0) RETURN
C
C  LOCATE A SUITABLE OR OPTIMAL EIGENVECTOR.
C
      CALL OPTEIG (XB,DB,G,ERR,MACHP,JA,N,NM)
      GO TO (20,30,50), ERR
C
C  ATTEMPT TO FIND A PERMUTATION THAT RELATES A AND B THROUGH USE OF AN
C  EIGENVECTOR THAT CORRESPONDS TO AN EIGENVALUE OF MULTIPLICITY ONE.
C
  20  STATUS=7
      CALL SINGLE (A,B,XA,XB,FA,FB,H,P,ERR,MACHP,MAXT,JA,N,NM)
      GO TO 40
C
C  ATTEMPT TO FIND A PERMUTATION THAT RELATES A AND B THROUGH USE OF AN
C  EIGENVECTOR THAT CORRESPONDS TO AN EIGENVALUE OF MULTIPLICITY TWO.
C
  30  STATUS=8
      CALL DOUBLE (A,B,C,XA,XB,CE,CF,FA,FB,H,P,ERR,MAXT,MACHP,JA,N,NM,NT
     +             )
  40  IF (ERR.NE.4) RETURN
C
C  COMPUTE AND COMPARE THE SPECTRA OF THE N ORDER N-1 SUBGRAPHS OF THE
C  ORIGINAL GRAPHS.
C
  50  STATUS=9
      CALL SUBVAL (A,B,C,XA,SC,SD,SE,H,P,ERR,MACHP,MAXT,N,NM)
      IF (ERR.NE.0.AND.ERR.NE.5+N) RETURN
C
C  ATTEMPT TO FIND A PERMUTATION THAT RELATES A AND B THROUGH USE OF AN
C  EIGENVECTOR THAT CORRESPONDS TO AN EIGENVALUE OF MULTIPLICITY ONE IN
C  A SUBGRAPH OF ORDER N-1.
C
      STATUS=10
      CALL SUBVEC (A,B,C,XA,XB,DA,DB,EA,EB,FA,FB,SC,SD,SE,G,H,P,ERR,
     +             MACHP,MAXT,N,NM)
C
      RETURN
      END
```

```
      SUBROUTINE LABEL (A,B,DA,DB,SC,ERR,N,NM)
C
C  THIS SUBROUTINE RECORDS AND COMPARES THE INITIAL LABELS OF THE NODES
C  OF THE TWO GRAPHS TO BE COMPARED.
C
C  ON INPUT:
C     A AND B CONTAIN THE TWO ADJACENCY MATRICES TO BE COMPARED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B ARE UNCHANGED.
C     DA AND DB CONTAIN THE DIAGONAL ELEMENTS -LABELS- OF A AND B.
C     ERR = 0 IF DA AND DB ARE ISOMORPHIC.  OTHERWISE ERR IS THE INDEX
C        OF THE ELEMENT OF DA WHICH CANNOT BE MATCHED IN DB.
C     N AND NM ARE UNCHANGED.
C
C  WORKING ARRAYS:   SC
C
      INTEGER N,NM,ERR,I
      REAL A(NM,NM),B(NM,NM),DA(NM),DB(NM),SC(NM)
C
C  RECORD THE LABELS.
C
      DO 10 I=1,N
         DA(I)=A(I,I)
         DB(I)=B(I,I)
   10 CONTINUE
C
C  COMPARE THE LABELS.
C  NOTE THAT SINCE THE LABELS ARE INTEGERS, THE MACHINE PRECISION
C  CONSTANT IS ZERO.
C
      CALL COMP (DA,DB,SC,ERR,0,N,NM)
C
      RETURN
      END
```

```
      SUBROUTINE COMP (RA,RB,RC,ERR,MACHP,N,NM)
C
C  THIS SUBROUTINE DETERMINES WHETHER OR NOT TWO VECTOR ARRAYS ARE
C  ISOMORPHIC.
C
C  ON INPUT:
C     RA AND RB ARE THE TWO VECTOR ARRAYS TO BE COMPARED.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     RA AND RB ARE UNCHANGED.
C     ERR = 0 IF RA AND RB ARE ISOMORPHIC.  OTHERWISE ERR IS THE INDEX
C        OF THE ELEMENT OF RA WHICH CANNOT BE MATCHED IN RB.
C     MACHP, N, AND NM ARE UNCHANGED.
C
C  WORKING ARRAYS:   RC
C
      INTEGER NM,N,ERR,I,J
      REAL RA(NM),RB(NM),RC(NM),MACHP,VAL
C
      VAL=1
      ERR=0
C
C  COPY RB INTO RC AND COMPUTE A RELATIVELY LARGE NUMBER.
C
      DO 10 I=1,N
         RC(I)=RB(I)
         VAL=VAL+ABS(RB(I))
   10 CONTINUE
C
C  COMPARE THE TWO ARRAYS.
C
      DO 30 I=1,N
         DO 20 J=1,N
            IF (ABS(RA(I)-RC(J)).GT.MACHP*(1+ABS(RC(J)))) GO TO 20
               RC(J)=VAL
               GO TO 30
   20       CONTINUE
               ERR=I
               RETURN
   30 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE VALENC (A,B,DA,DB,SC,ERR,N,NM)
C
C  THIS SUBROUTINE COMPUTES AND COMPARES THE NODE VALENCES OF THE
C  TWO GRAPHS.
C
C  ON INPUT:
C     A AND B CONTAIN THE TWO ADJACENCY MATRICES TO BE COMPARED.
C     DA AND DB CONTAIN THE NODE LABELS OF A AND B.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B ARE UNCHANGED.
C     DA AND DB CONTAIN THE UPDATED NODE LABELS.
C     ERR = 0 IF DA AND DB ARE ISOMORPHIC.  OTHERWISE ERR IS THE INDEX
C        OF THE ELEMENT OF DA WHICH CANNOT BE MATCHED IN DB.
C     N AND NM ARE UNCHANGED.
C
C  WORKING ARRAYS:   SC
C
      INTEGER NM,N,ERR,I,J
      REAL A(NM,NM),B(NM,NM),DA(NM),DB(NM),SC(NM)
C
      DO 20 I=1,N
C
C  MAKE ROOM FOR NEW INFORMATION.
C
         DA(I)=100*DA(I)
         DB(I)=100*DB(I)
C
C  COMPUTE AND RECORD NODE VALENCES - UPDATE LABELS.
C
         DO 10 J=1,N
            IF (I.EQ.J) GO TO 10
               DA(I)=DA(I)+A(I,J)
               DB(I)=DB(I)+B(I,J)
   10    CONTINUE
   20 CONTINUE
C
C  COMPARE UPDATED LABELS.
C
      CALL COMP (DA,DB,SC,ERR,0,N,NM)
C
      RETURN
      END
```

```
      SUBROUTINE DUPLIC (A,B,DA,DB,SC,SD,ERR,Q,N,NM)
C
C  THIS SUBROUTINE LOCATES AND DISTINGUISHES DUPLICATE NODES THROUGH
C  LABEL MODIFICATION.
C
C  ON INPUT:
C     A AND B CONTAIN THE TWO ADJACENCY MATRICES TO BE COMPARED.
C     DA AND DB CONTAIN CURRENT NODE LABEL DATA.
C     Q = 4 IF 0'S ARE TO BE PLACED ON THE DIAGONAL OF A AND B, AND
C        Q = 5 IF 1'S ARE TO BE USED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B HAVE DIAGONAL ENTRIES OF ALL 0'S OR ALL 1'S BUT ARE
C        OTHERWISE UNCHANGED.
C     DA AND DB CONTAIN UPDATED LABEL DATA.
C     ERR = 0 IF DA AND DB ARE ISOMORPHIC.  OTHERWISE ERR IS THE INDEX
C        OF THE ELEMENT OF DA WHICH CANNOT BE MATCHED IN DB.
C     Q, N, AND NM ARE UNCHANGED.
C
C  WORKING VARIABLES:   SC,SD
C
      INTEGER NM,N,NN,I,J,K,L,ERR,Q
      REAL A(NM,NM),B(NM,NM),DA(NM),DB(NM),SC(NM),SD(NM)
C
      NN=N-1
C
      DO 10 I=1,N
C
C  SET DIAGONAL ELEMENTS TO 0 OR 1.
C
         A(I,I)=Q-4
         B(I,I)=Q-4
C
C  MAKE ROOM FOR NEW DATA.
C
         DA(I)=100*DA(I)
         DB(I)=100*DB(I)
C
C  STORE DA IN SC AND DB IN SD.
C
         SC(I)=DA(I)
         SD(I)=DB(I)
   10 CONTINUE
C
      DO 60 I=1,NN
         L=I+1
         DO 50 K=L,N
C
C  SEARCH FOR IDENTICAL ROWS IN A.
C
            IF (SC(I).NE.SC(K)) GO TO 30
               DO 20 J=1,N
                  IF (A(I,J).NE.A(K,J)) GO TO 30
   20          CONTINUE
                  DA(K)=DA(K)+1
                  DA(I)=DA(I)+2
```

```
C
C  SEARCH FOR IDENTICAL ROWS IN B.
C
  30         IF (SD(I).NE.SD(K)) GO TO 50
                DO 40 J=1,N
                   IF (B(I,J).NE.B(K,J)) GO TO 50
  40            CONTINUE
                   DB(K)=DB(K)+1
                   DB(I)=DB(I)+2
  50      CONTINUE
  60   CONTINUE
C
C  COMPARE UPDATED LABELS.
C
       CALL COMP (DA,DB,SC,ERR,0,N,NM)
C
       RETURN
       END
```

```
      SUBROUTINE PREPAR (A,B,DA,DB,N,NM)
C
C  THIS SUBROUTINE CONVERTS THE ADJACENCY MATRIX FROM 0,1 FORMAT TO 1,X
C  FORMAT.  IN ADDITION THE LABELS ARE CONVERTED INTO MULTIPLES OF N
C  AND ARE ENTERED AS DIAGONAL ELEMENTS OF A AND B.
C
C  ON INPUT:
C     A AND B ARE THE ADJACENCY MATRICES IN 0,1 FORMAT.
C     DA AND DB CONTAIN CURRENT NODE LABEL DATA.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B CONTAIN THE TWO ADJACENCY MATRICES IN 1,X FORM WITH
C        MODIFIED DIAGONAL ELEMENTS IN MULTIPLES OF N.
C     DA AND DB CONTAIN THE NODE LABEL DATA IN THE FORM OF MULTIPLES
C        OF N.
C     N AND NM ARE UNCHANGED.
C
      INTEGER NM,N,L
      REAL A(NM,NM),B(NM,NM),DA(NM),DB(NM),FN,NA,NB,QA,QB,INF
C
C  FN IS THE FUNCTION FOR X-1 IN THE 1,X FORM OF THE ADJACENCY MATRIX.
C
      FN=N/2
      QA=-1
      QB=-1
      NA=0
      NB=0
      INF=1000000000
C
C  CONVERT A AND B TO 1,X FORM.
C
      DO 20 I=1,N
         DO 10 J=1,N
            A(I,J)=A(I,J)*FN+1
            B(I,J)=B(I,J)*FN+1
   10    CONTINUE
   20 CONTINUE
C
      DO 50 I=1,N
C
C  LABEL DIAGONAL ELEMENTS OF A IN MULTIPLES OF N.
C
         L=1
         DO 30 J=1,N
            IF (DA(J).LT.DA(L)) L=J
   30    CONTINUE
         IF (DA(L).GT.QA) NA=NA+N
         A(L,L)=NA
         QA=DA(L)
         DA(L)=INF
C
C  LABEL DIAGONAL ELEMENTS OF B IN MULTIPLES OF N.
C
         L=1
         DO 40 J=1,N
            IF (DB(J).LT.DB(L)) L=J
```

```
   40      CONTINUE
           IF (DB(L).GT.QB) NB=NB+N
           B(L,L)=NB
           QB=DB(L)
           DB(L)=INF
   50   CONTINUE
C
C  RESTORE DA AND DB.
C
        DO 60 I=1,N
           DA(I)=A(I,I)
           DB(I)=B(I,I)
   60   CONTINUE
C
        RETURN
        END
```

```
      SUBROUTINE SPECTR (A,B,XA,XB,EA,EB,SC,G,ERR,MACHP,N,NM)
C
C  THIS SUBROUTINE CALCULATES AND COMPARES THE SPECTRA OF THE TWO
C  MODIFIED ADJACENCY MATRICES.  IN ADDITION, DATA PERTAINING TO THE
C  MULTIPLICITY OF THE EIGENVALUES IS RECORDED.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B ARE UNCHANGED.
C     XA AND XB CONTAIN THE EIGENVECTORS OF A AND B IN THE SAME ORDER
C        AS THE CORRESPONDING EIGENVALUES.
C     EA AND EB CONTAIN THE EIGENVALUES OF A AND B IN NONDECREASING
C        ORDER.
C     G OF I IS 1 IF THE I TH AND I+1 TH EIGENVALUES OF A ARE IDENTICAL
C        AND IS 0 OTHERWISE.
C     ERR IS NEGATIVE IF AN ERROR CONDITION WAS RAISED IN THE EISPACK
C        SOFTWARE.  ERR IS 0 IF THE SPECTRA ARE IDENTICAL.  OTHERWISE,
C        ERR IS THE INDEX OF THE EIGENVALUE OF A THAT COULD NOT BE
C        MATCHED WITH AN EIGENVALUE OF B.
C     MACHP, N, AND NM ARE UNCHANGED.
C
C  WORKING VARIABLES:    SC
C
      INTEGER NM,N,ERR,I,NN,G(NM)
      REAL A(NM,NM),B(NM,NM),XA(NM,NM),XB(NM,NM),EA(NM),EB(NM),SC(NM),
     +     MACHP
C
      NN=N-1
C
C  FIND THE EIGENVALUES AND EIGENVECTORS OF A.
C
      CALL TRED2 (NM,N,A,EA,SC,XA)
      CALL TQL2 (NM,N,EA,SC,XA,ERR)
      IF (ERR.NE.0) GO TO 10
C
C  FIND THE EIGENVALUES AND EIGENVECTORS OF B.
C
         CALL TRED2 (NM,N,B,EB,SC,XB)
         CALL TQL2 (NM,N,EB,SC,XB,ERR)
  10  ERR=-ERR
      IF (ERR.EQ.0) CALL COMP (EA,EB,SC,ERR,MACHP,N,NM)
C
C  RECORD EIGENVALUE MULTIPLICITY DATA.
C
      IF (ERR.NE.0) RETURN
      DO 20 I=1,NN
         G(I)=0
         IF (ABS(EA(I)-EA(I+1)).LT.MACHP*(1+ABS(EA(I)))) G(I)=1
  20  CONTINUE
      G(N)=0
C
      RETURN
      END
```

```
      SUBROUTINE OPTEIG (XB,DB,G,ERR,MACHP,JA,N,NM)
C
C  THIS SUBROUTINE SELECTS AN OPTIMAL EIGENVECTOR FOR USE IN GENERATING
C  A SERIES OF PERMUTATIONS.
C
C  ON INPUT:
C     XB IS THE EIGENVECTOR MATRIX OF B.
C     DB CONTAINS THE DIAGONAL ELEMENTS OF B.
C     G CONTAINS EIGENVALUE MULTIPLICITY INFORMATION.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     XB IS ESSENTIALLY UNCHANGED.  THE POSSIBILITY EXISTS THAT ANY TWO
C        EIGENVECTORS CORRESPONDING TO THE SAME EIGENVALUE COULD BE
C        INTERCHANGED.
C     DB AND G ARE UNCHANGED.
C     ERR = 1 IF THE EIGENVECTOR SELECTED CORRESPONDS TO AN EIGENVALUE
C        OF MULTIPLICITY 1.  ERR=2 IF THE CORRESPONDING EIGENVALUE HAS
C        MULTIPLICITY 2.  ERR=3 IF NO SUITABLE EIGENVECTOR COULD BE
C        FOUND.
C     MACHP IS UNCHANGED.
C     JA IS THE COLUMN INDEX OF THE OPTIMAL EIGENVECTOR, IF ONE EXISTS.
C     N AND NM ARE UNCHANGED.
C
      INTEGER NM,N,I,J,PN,LT,PX,ERR,JA,LI,LM,L,NN,G(NM)
      REAL XB(NM,NM),DB(NM),MACHP
C
      NN=N-1
      ERR=3
      PN=1000
C
C  ALL EIGENVECTORS WITH THREE OR MORE IDENTICAL COMPONENTS OR THAT
C  CORRESPOND TO EIGENVALUES OF MULTIPLICITY THREE OR MORE ARE
C  ELIMINATED FROM CONSIDERATION.  EIGENVECTORS THAT CORRESPOND TO
C  EIGENVALUES OF MULTIPLICITY ONE ARE CONSIDERED PREFERENTIALLY TO
C  THOSE WHICH HAVE MULTIPLICITY TWO.  EIGENVECTORS WHICH HAVE FEW PAIRS
C  OF IDENTICAL COMPONENTS ARE FAVORED OVER THOSE WITH MANY SUCH PAIRS.
C
      DO 80 J=1,N
         LT=0
         PX=1
C
C  CHECK FOR MULTIPLICITY OF ASSOCIATED EIGENVALUE.
C
         IF (J.EQ.1) GO TO 20
            IF (G(J-1).EQ.0) GO TO 20
               IF (J.EQ.2) GO TO 10
                  IF (G(J-2).EQ.1.) GO TO 80
10                IF (G(J).EQ.1) GO TO 80
                     LT=4
                     PX=4
                     GO TO 30
20       IF (G(J).EQ.0) GO TO 30
            IF (G(J+1).EQ.1) GO TO 80
               LT=4
               PX=2
```

```
C
C  CHECK FOR IDENTICAL COMPONENTS.
C
  30      DO 50 I=1,NN
             LI=I+1
             LM=0
             DO 40 L=LI,N
                IF (ABS(XB(I,J)-XB(L,J)+DB(I)-DB(L)).GT.MACHP) GO TO 40
                   LT=LT+1
                   IF (LM.EQ.1) GO TO 80
                      LM=1
  40         CONTINUE
  50      CONTINUE
          IF (LT.GE.PN) GO TO 80
             JA=J
             ERR=PX
             IF (ERR.LT.4) GO TO 70
                ERR=2
                JA=JA-1
C
C  INTERCHANGE TWO EIGENVECTORS THAT CORRESPOND TO THE SAME EIGENVALUE
C  SO THAT IN THE RESULTING PAIR THE FIRST WILL BE MORE SUITABLE THAN
C  THE LAST.
C
                DO 60 I=1,N
                   TEMP=XB(I,JA)
                   XB(I,JA)=XB(I,JA+1)
                   XB(I,JA+1)=TEMP
  60            CONTINUE
  70         PN=LT
C
C  IF EIGENVECTOR IS SUITABLE -NOT NECESSARILY OPTIMAL- RETURN.
C
             IF (LT.LT.4) RETURN
  80  CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE SINGLE (A,B,XA,XB,FA,FB,H,P,ERR,MACHP,MAXT,JA,N,NM)
C
C  THIS SUBROUTINE GENERATES AND TESTS UP TO MAXT PERMUTATIONS FROM DATA
C  FOUND IN AN EIGENVECTOR CORRESPONDING TO AN EIGENVALUE OF
C  MULTIPLICITY ONE.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     XA AND XB ARE THE EIGENVECTOR MATRICES OF A AND B THOUGH ONLY THE
C        JA TH COLUMNS OF XA AND XB ARE USED IN THIS SUBROUTINE.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     MAXT IS THE MAXIMUM NUMBER OF PERMUTATIONS THAT CAN BE TESTED.
C     JA IS THE COLUMN OF XA AND XB WHICH CONTAINS THE EIGENVECTORS TO
C        BE EXAMINED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A, B, XA, AND XB ARE UNCHANGED.
C     P CONTAINS THE LAST PERMUTATION THAT WAS CHECKED AGAINST A AND B.
C        IF ERR=0, THIS PERMUTATION RELATES A AND B.
C     ERR = -1 IF NO SUITABLE PERMUTATIONS COULD BE GENERATED AND THE
C        GRAPHS ARE NOT ISOMORPHIC.  ERR=1 IF THREE IDENTICAL COMPONENTS
C        WERE FOUND IN THE EIGENVECTOR.  ERR=3 IF ALL POSSIBLE
C        PERMUTATIONS FAILED WHEN TESTED AGAINST A AND B AND THE GRAPHS
C        ARE NOT ISOMORPHIC.  ERR=4 IF THE MAXIMUM NUMBER OF
C        PERMUTATIONS WERE TRIED WITHOUT SUCCESS.  ERR=0 IF A
C        PERMUTATION WAS FOUND TO RELATE A AND B.
C     MACHP, MAXT, JA, N, AND NM ARE UNCHANGED.
C
C  WORKING VARIABLES:   FA,FB,H
C
      INTEGER NM,N,ERR,JA,I,M,H(NM),P(NM),LT,MAXT
      REAL A(NM,NM),B(NM,NM),XA(NM,NM),XB(NM,NM),FA(NM),FB(NM),MACHP
C
      M=0
C
C  USE THE NODE LABELS TO AIDE IN THE SEPARATION OF IDENTICAL
C  COMPONENTS.
C
      DO 10 I=1,N
         FA(I)=A(I,I)+XA(I,JA)
         FB(I)=B(I,I)+XB(I,JA)
  10  CONTINUE
C
C  GENERATE A POTENTIAL PERMUTATION AS WELL AS THE DATA NECESSARY TO
C  FIND ALL OTHER POSSIBLE PERMUTATIONS.
C
  20  CALL PERM (FA,FB,H,P,ERR,MACHP,LT,N,NM)
      IF (ERR.EQ.1) RETURN
         IF (ERR.EQ.-1) GO TO 30
C
C  TEST UP TO MAXT PERMUTATIONS AGAINST A AND B.
C
            CALL TEST (A,B,H,P,ERR,MAXT,LT,N,NM)
            IF (ERR.NE.3) RETURN
  30  IF (M.EQ.1) RETURN
C
```

```
C  SHOULD ALL PERMUTATIONS FAIL, REPEAT THE PROCESS WITH ONE EIGENVECTOR
C  NEGATED TO INCLUDE ALL POSSIBLE CASES.
C
         DO 40 I=1,N
            FB(I)=B(I,I)-XB(I,JA)
  40     CONTINUE
         M=1
         GO TO 20
C
      END
```

```
      SUBROUTINE PERM (FA,FB,H,P,ERR,MACHP,LT,N,NM)
C
C  THIS SUBROUTINE GENERATES A SINGLE PERMUTATION BETWEEN THE COMPONENTS
C  OF TWO VECTORS AND RECORDS THE DATA NECESSARY TO GENERATE ALL OTHER
C  POSSIBLE PERMUTATIONS BETWEEN THE COMPONENTS OF THE TWO VECTORS.  THE
C  VECTORS MUST NOT CONTAIN MORE THAN TWO IDENTICAL COMPONENTS.
C
C  ON INPUT:
C     FA AND FB ARE THE TWO VECTORS FROM WHICH ALL PERMUTATIONS WILL
C        BE GENERATED.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     FA IS UNCHANGED.
C     FB IS DESTROYED.
C     H CONTAINS THE INDICES OF THE PAIRS OF IDENTICAL COMPONENTS OF FA.
C     P CONTAINS A PERMUTATION OF THE COMPONENTS OF FA INTO THOSE OF FB.
C     ERR = -1 IF THE TWO VECTORS ARE FOUND TO BE NON-ISOMORPHIC.
C        ERR=1 IF THREE OR MORE COMPONENTS OF FA ARE IDENTICAL.
C        ERR=0 OTHERWISE.
C     MACHP IS UNCHANGED.
C     LT IS THE NUMBER OF IDENTICAL PAIRS OF COMPONENTS FOUND IN FA.
C     N AND NM ARE UNCHANGED.
C
      INTEGER NM,N,I,J,L,LT,H(NM),P(NM),ERR
      REAL FA(NM),FB(NM),MACHP,INF
C
      ERR=1
      LT=0
      INF=-10
C
      DO 70 I=1,N
         L=-1
C
C  FIND ALL COMPONENTS OF FB THAT MATCH THE CURRENT COMPONENT OF FA.
C
         DO 30 J=1,N
            IF (ABS(FA(I)-FB(J)).GT.MACHP) GO TO 30
               IF (L) 10,20,80
 10               FB(J)=INF
                  P(I)=J
 20            L=L+1
 30      CONTINUE
C
         IF (L) 40,70,50
 40         ERR=-1
            RETURN
C
C  RECORD THE INDEX OF THE FIRST HALF OF A IDENTICAL PAIR IN FA.
C
 50         LT=LT+1
            H(2*LT-1)=I
C
C  FIND AND RECORD THE INDEX OF THE SECOND HALF OF THE IDENTICAL PAIR
C  IN FA.
C
```

```
            DO 60 J=1,N
               IF (ABS(FA(I)-FA(J)).GT.MACHP.OR.I.EQ.J) GO TO 60
                  H(2*LT)=J
                  GO TO 70
   60       CONTINUE
   70 CONTINUE
C
      ERR=0
   80 RETURN
C
      END
```

```
      SUBROUTINE TEST (A,B,H,P,ERR,MAXT,LT,N,NM)
C
C  THIS SUBROUTINE TESTS UP TO MAXT PERMUTATIONS AGAINST THE ORIGINAL
C  GRAPHS.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     H CONTAINS THE PAIRS OF INDICES OF P WHICH MUST BE INTERCHANGED
C        BEFORE ALL POSSIBLE PERMUTATIONS CAN BEEN TESTED.
C     P CONTAINS THE FIRST PERMUTATION TO BE CHECKED.
C     MAXT IS THE MAXIMUM NUMBER OF PERMUTATIONS WHICH MAY BE TESTED.
C     LT IS THE NUMBER OF PAIRS OF INDICES OF P THAT MAY HAVE TO BE
C        INTERCHANGED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A, B, AND H ARE UNCHANGED.
C     P CONTAINS THE LAST PERMUTATION THAT WAS CHECKED AGAINST A AND B.
C        IF ERR=0, THEN THIS PERMUTATION RELATES A AND B.
C     ERR = 0 IF A PERMUTATION WAS SUCCESSFUL IN RELATING A AND B.
C        ERR=3 IF ALL PERMUTATIONS FAILED WHEN TESTED AGAINST A AND B.
C        ERR=4 IF MAXT PERMUTATIONS WERE TRIED WITHOUT SUCCESS.
C     LT, N, AND NM ARE UNCHANGED.
C
      INTEGER NM,N,LT,ERR,LB,I,J,BIN,MAXT,T,L,K,H(NM),P(NM),M
      REAL A(NM,NM),B(NM,NM)
C
C  LB IS THE MAXIMUM NUMBER LESS 1 OF PERMUTATIONS WHICH MAY HAVE TO BE
C  TESTED.
C
      LB=2**LT-1
C
C  BIN CONTROLS WHICH PAIRS OF INDICES ARE TO BE INTERCHANGED EACH TIME.
C
      BIN=0
      T=0
      ERR=0
C
   10 T=T+1
C
C  TEST THE PERMUTATION IN P AGAINST A AND B.
C
      DO 30 I=1,N
         L=P(I)
         DO 20 J=1,N
            K=P(J)
            IF (B(L,K).NE.A(I,J)) GO TO 40
   20    CONTINUE
   30 CONTINUE
      RETURN
C
C  CHECK IF ALL POSSIBLE PERMUTATIONS HAVE BEEN TRIED.
C
   40 IF (BIN.LT.LB) GO TO 50
         ERR=3
         RETURN
C
```

```
C  CHECK IF MAXT PERMUTATIONS HAVE BEEN TRIED.
C
  50  IF (T.LT.MAXT) GO TO 60
         ERR=4
         RETURN
C
C  GENERATE THE NEXT PERMUTATION TO BE TESTED.
C
  60  BIN=BIN+1
C
C  SEQUENTIALLY INTERCHANGE I+1 PAIRS OF VALUES OF P WHERE 2**I IS THE
C  LARGEST POWER OF 2 THAT DIVIDES BIN.  ON THE AVERAGE, TWO PAIRS WILL
C  BE INTERCHANGED EACH TIME, INDEPENDENT OF THE VALUE OF LT.
C
      M=1
      DO 70 I=1,LT
         L=H(2*I-1)
         K=H(2*I)
         J=P(L)
         P(L)=P(K)
         P(K)=J
         M=2*M
         IF (MOD(BIN,M).GT.0) GO TO 10
  70  CONTINUE
C
      END
```

```
      SUBROUTINE DOUBLE (A,B,C,XA,XB,CE,CF,FA,FB,H,P,ERR,MAXT,MACHP,JF,
     +                   N,NM,NT)
C
C  THIS SUBROUTINE GENERATES AND TESTS PERMUTATIONS FROM DATA FOUND IN
C  EIGENVECTORS CORRESPONDING TO EIGENVALUES OF MULTIPLICITY TWO.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     XA AND XB ARE THE EIGENVECTOR MATRICES OF A AND B.
C     MAXT IS THE MAXIMUM NUMBER OF PERMUTATIONS THAT COULD BE TRIED IF
C        THE OPTIMAL EIGENVECTOR CORRESPONDED TO AN EIGENVALUE OF
C        MULTIPLICTY ONE.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     JF AND JF+1 ARE THE TWO COLUMNS OF XA IN WHICH THE PERTINENT
C        EIGENVECTORS ARE STORED.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF MOST OF THE ARRAYS.
C     NT = 2*NM AND IS THE DIMENSION OF CE AND CF.
C
C  ON RETURN:
C     A, B, XA, AND XB ARE UNCHANGED.
C     P CONTAINS THE LAST PERMUTATION THAT WAS CHECKED AGAINST A AND B.
C        IF ERR=0, THIS PERMUTATION RELATES A AND B.
C     ERR = 4 IF NO PERMUTATION WAS FOUND TO RELATE A AND B BUT NOT ALL
C        POSSIBLE PERMUTATIONS WERE TESTED DUE TO THE LIMIT IMPOSED
C        BY MAXT AND MAXTN.  ERR=0 IF A PERMUTATION SUCCESSFULLY RELATED
C        A AND B.  ERR=-1,1, OR 3 IF ALL POSSIBLE PERMUTATIONS FAILED
C        WHEN TESTED AGAINST A AND B AND THE GRAPHS ARE NOT ISOMORPHIC.
C     MAXT, MACHP, JA, N, NM, AND NT ARE UNCHANGED.
C
C  WORKING VARIABLES:   C,CE,CF,FA,FB,H
C
      INTEGER NM,N,ERR,K,I,J,M,JF,NT,L,MAXT,MAXTN,H(NM),P(NM)
      REAL A(NM,NM),B(NM,NM),C(NM,NM),XA(NM,NM),XB(NM,NM),FA(NM),FB(NM),
     +     CE(NT),CF(NT),MACHP,MACHPN,TM
C
C  RELAX THE REQUIREMENT FOR EQUALITY AS MULTIPLE OPERATIONS ARE TO BE
C  PERFORMED ON THE EIGENVECTOR COMPONENTS.
C
      MACHPN=N*MACHP/5
C
C  SET THE MAXIMUM NUMBER OF TESTS THAT MAY BE PERFORMED ON EACH
C  PROPOSED LINEAR COMBINATION OF THE TWO EIGENVECTORS.
C
      MAXTN=4*MAXT/N
C
      DO 10 J=1,N
         L=J
         IF (ABS(XA(J,JF)).GT.MACHPN.OR.ABS(XA(J,JF+1)).GT.MACHPN)
     +                   GO TO 20
   10 CONTINUE
C
C  COMPUTE ALL POSSIBLE VALUES OF THE TWO COEFFICIENTS AND RECORD THEM
C  IN CE AND CF.
C
   20 TM=XA(L,JF)**2+XA(L,JF+1)**2
      DO 50 J=1,N
         K=2*J
```

```
      CE(K-1)=(XA(L,JF)*XB(J,JF)+XA(L,JF+1)*XB(J,JF+1))/TM
      CF(K-1)=(XA(L,JF+1)*XB(J,JF)-XA(L,JF)*XB(J,JF+1))/TM
      CE(K)=(XA(L,JF)*XB(J,JF)-XA(L,JF+1)*XB(J,JF+1))/TM
      CF(K)=(XA(L,JF+1)*XB(J,JF)+XA(L,JF)*XB(J,JF+1))/TM
C
C  AVOID UNNECESSARY WORK THROUGH DELETION OF DUPLICATED COEFFICIENTS.
C
      IF (J.EQ.1) GO TO 40
         M=K-2
         DO 30 I=1,M
            IF (ABS(CE(I)-CE(K-1)).LT.MACHP) CE(K-1)=2
            IF (ABS(CE(I)-CE(K)).LT.MACHP) CE(K)=2
30       CONTINUE
40    IF (ABS(CE(K)-CE(K-1)).LT.MACHP) CE(K)=2
C
50  CONTINUE
C
C  TEST ALL POSSIBLE COEFFICIENTS.
C
    L=0
    DO 80 J=1,NT
C
C  ELIMINATE FROM CONSIDERATION COEFFICIENTS WITH ABSOLUTE VALUE > 1.
C
      IF (ABS(CE(J)).GT.1+MACHPN) GO TO 80
      IF (ABS(CE(J)).GT.1) CE(J)=1
C
C  PERFORM THE LINEAR COMBINATION TO CREATE A PSEUDO EIGENVECTOR.
C
      DO 70 I=1,N
         C(I,JF)=CE(J)*XA(I,JF)+CF(J)*XA(I,JF+1)
70    CONTINUE
C
C  TREAT THE PSEUDO EIGENVECTOR AS ONE THAT CORRESPONDS TO AN EIGENVALUE
C  OF MULTIPLICITY ONE AND CHECK ALL POSSIBLE PERMUTATIONS.
C
      CALL SINGLE (A,B,C,XB,FA,FB,H,P,ERR,MACHPN,MAXTN,JF,N,NM)
      IF (ERR.EQ.0) RETURN
      IF (ERR.EQ.4) L=4
80  CONTINUE
    IF (L.EQ.4) ERR=4
C
    RETURN
    END
```

```
      SUBROUTINE SUBVAL (A,B,C,XA,SC,SD,SE,H,P,ERR,MACHP,MAXT,N,NM)
C
C  THIS SUBROUTINE GENERATES AND COMPARES THE SPECTRA OF THE ORDER N-1
C  SUBGRAPHS OF THE TWO ORIGINAL GRAPHS.  AN ATTEMPT IS MADE TO GENERATE
C  A PERMUTATION BETWEEN THE TWO GRAPHS BASED ON THIS DATA.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     MAXT IS THE MAXIMUM NUMBER OF PERMUTATIONS THAT COULD BE TRIED IF
C        THE OPTIMAL EIGENVECTOR CORRESPONDED TO AN EIGENVALUE OF
C        MULTIPLICTY ONE.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B ARE UNCHANGED.
C     C CONTAINS SPECTRA DUPLICATION INFORMATION.  THE I TH ROW OF C
C        CONTAINS THE INDECIES OF ALL NODES IN A WHICH WHEN UNIQUELY
C        MARKED PRODUCE SPECTRA IDENTICAL TO THAT PRODUCED WHEN THE I TH
C        NODE OF A WAS UNIQUELY MARKED.
C     XA CONTAINS THE N SPECTRA GENERATED FROM THE SUBGRAPHS OF A.  EACH
C        ROW CONTAINS ONE SPECTRA.
C     SC OF I IS THE NUMBER OF NODES IN A, INCLUDING I, THAT PRODUCED
C        SPECTRA IDENTICAL TO THAT OF I.
C     P CONTAINS A PERMUTATION OF THE SET OF SPECTRA OF SUBGRAPHS OF A
C        INTO THOSE OF B.  IF ERR=N+1, THIS IS A PERMUTATION OF A
C        INTO B.
C     ERR IS NEGATIVE IF AN ERROR OCCURS IN THE EISPACK SOFTWARE.  IF
C        THE SETS OF SPECTRA ARE NOT ISOMORPHIC, ERR CONTAINS THE INDEX
C        OF THE NODE IN B WHICH WHEN UNIQUELY MARKED, PRODUCED A SPECTRA
C        THAT COULD NOT BE MATCHED WITH A SPECTRA FROM THE SUBGRAPHS OF
C        A.  ERR=0 IF THE SETS OF SPECTRA WERE ISOMORPHIC BUT THREE OR
C        MORE NODES OF A, WHICH WHEN UNIQUELY MARKED, PRODUCED IDENTICAL
C        SPECTRA.  ERR=N+1 IF A PERMUTATION RELATING A AND B WAS FOUND.
C        ERR=N+4 IF ALL POSSIBLE PERMUTATIONS FAILED TO RELATE A AND B.
C        ERR=N+5 IF MAXT PERMUTATIONS WERE GENERATED AND TESTED WITHOUT
C        SUCCESS.
C     MACHP, MAXT, N, AND NM ARE UNCHANGED.
C
C  WORKING VARIABLES:    SD,SE,H
C
      INTEGER NM,N,I,J,K,L,ERR,M,MAXT,LT,H(NM),P(NM)
      REAL A(NM,NM),B(NM,NM),C(NM,NM),XA(NM,NM),SC(NM),SD(NM),SE(NM),FN,
     +     MACHP
C
C  FN IS THE FUNCTION FOR X-1 IN THE 1,X FORM OF THE ADJACENCY MATRIX.
C
      FN=N/2
C
C  GENERATE THE SPECTRA OF ALL ORDER N-1 SUBGRAPHS OF A.
C
      DO 50 I=1,N
         P(I)=0
C
C  COPY A INTO C.
C
         DO 20 K=1,N
```

```
            DO 10 J=1,N
               C(K,J)=A(K,J)
 10         CONTINUE
 20      CONTINUE
C
C  UNIQUELY MARK THE I TH NODE OF C=A AND ALL NODES CONNECTED TO IT.
C
         DO 30 J=1,N
            C(J,J)=C(J,J)+C(I,J)
            C(I,J)=3*FN
            C(J,I)=3*FN
 30      CONTINUE
         C(I,I)=C(I,I)+FN
C
C  COMPUTE THE SPECTRA  -  C IS DESTROYED IN THIS COMPUTATION.
C
         CALL TRED1 (NM,N,C,SC,SD,SE)
         CALL TQL1 (N,SC,SD,ERR)
         ERR=-ERR
         IF (ERR.NE.0) RETURN
         DO 40 J=1,N
            XA(I,J)=SC(J)
 40      CONTINUE
 50   CONTINUE
C
C  SEQUENTIALLY COMPUTE THE SPECTRA OF THE N-1 ORDER SUBGRAPHS OF B,
C  CHECKING THAT EACH CAN BE MATCHED WITH A SIMILAR SPECTRA FROM THE
C  SUBGRAPHS OF A.
C
      DO 110 I=1,N
         DO 70 K=1,N
            DO 60 J=1,N
               C(K,J)=B(K,J)
 60         CONTINUE
 70      CONTINUE
         DO 80 J=1,N
            C(J,J)=C(J,J)+C(I,J)
            C(I,J)=3*FN
            C(J,I)=3*FN
 80      CONTINUE
         C(I,I)=C(I,I)+FN
         CALL TRED1 (NM,N,C,SC,SD,SE)
         CALL TQL1 (N,SC,SD,ERR)
         ERR=-ERR
         IF (ERR.NE.0) RETURN
         DO 100 J=1,N
            IF (P(J).GT.0) GO TO 100
               DO 90 K=1,N
                  IF (ABS(XA(J,K)-SC(K)).GT.MACHP*(1+ABS(SC(K))))
     +                              GO TO 100
 90               CONTINUE
                  P(J)=I
                  GO TO 110
100      CONTINUE
         ERR=I
         RETURN
110   CONTINUE
C
```

```
      LT=0
      L=0
C
C  COMPUTE AND RECORD THE DATA PERTAINING TO SPECTRA DUPLICATION IN A.
C
      DO 140 I=1,N
         M=0
         DO 130 J=1,N
            DO 120 K=1,N
               IF (ABS(XA(I,K)-XA(J,K)).GT.MACHP*(1+ABS(XA(I,K))))
     +                GO TO 130
  120       CONTINUE
               M=M+1
               C(I,M)=P(J)
               IF (M.EQ.3) L=1
               IF (L.EQ.1.OR.I.GE.J) GO TO 130
C
C  RECORD DUPLICATED PAIRS OF SPECTRA WITH THE HOPE THAT THERE ARE NO
C  TRIPLICATES AND SUBROUTINE TEST CAN BE USED TO FIND A PERMUTATION OF
C  A INTO B OR PROVE THAT ONE DOES NOT EXIST.
C
                  LT=LT+1
                  H(2*LT-1)=I
                  H(2*LT)=J
  130      CONTINUE
           SC(I)=M
  140   CONTINUE
C
      IF (L.EQ.1) RETURN
      CALL TEST (A,B,H,P,ERR,MAXT,LT,N,NM)
      ERR=ERR+N+1
C
      RETURN
      END
```

```
      SUBROUTINE SUBVEC (A,B,C,XA,XB,DA,DB,EA,EB,FA,FB,SC,SD,SE,G,H,P,
     +                   ERR,MACHP,MAXT,N,NM)
C
C  THIS SUBROUTINE ATTEMPTS TO GENERATE A PERMUTATION BETWEEN A AND B
C  THROUGH EXAMINATION OF EIGENVECTORS CORRESPONDING TO EIGENVALUES OF
C  MULTIPLICITY ONE OF ORDER N-1 SUBGRAPHS OF A AND B.
C
C  ON INPUT:
C     A AND B CONTAIN THE ADJACENCY MATRICES OF THE TWO GRAPHS.
C     C AND SC CONTAIN SPECTRA DUPLICATION INFORMATION.
C     P CONTAINS A PERMUTATION OF THE SET OF ORDER N-1 SPECTRA OF A INTO
C        THOSE OF B.
C     MACHP IS THE PRECISION WITHIN WHICH EQUALITY IS TO BE TESTED.
C     MAXT IS THE MAXIMUM NUMBER OF PERMUTATIONS THAT COULD BE TRIED IF
C        THE OPTIMAL EIGENVECTOR CORRESPONDED TO AN EIGENVALUE OF
C        MULTIPLICTY ONE.
C     N IS THE ORDER OF THE GRAPHS.
C     NM IS THE DIMENSION OF THE ARRAYS.
C
C  ON RETURN:
C     A AND B ARE UNCHANGED.
C     C AND SC ARE DESTROYED.
C     P CONTAINS A PERMUTATION OF THE COMPONENTS OF AN EIGENVECTOR OF
C        A SUBGRAPH OF A INTO AN EIGENVECTOR OF A SUBGRAPH OF B.  IF
C        ERR=0 THEN THE PERMUTATION RELATES A AND B.
C     ERR = 0 IF A PERMUTATION OF A INTO B WAS FOUND.  ERR=1 IF ALL
C        POSSIBLE PERMUTATIONS FAILED TO RELATE A AND B.  ERR=2 IF ALL
C        PERMUTATIONS CHECKED FAILED BUT NOT ALL POSSIBLE PERMUTATIONS
C        WERE TESTED DUE TO THE RESTRICTIONS IMPOSED BY MAXT.  ERR=3 IF
C        ALL EIGENVECTORS CORRESPONDING TO EIGENVALUES OF MULTIPLICITY
C        ONE HAD THREE OR MORE IDENTICAL COMPONENTS.  ERR IS NEGATIVE IF
C        AN ERROR CONDITION WAS RAISED IN THE EISPACK SOFTWARE.
C     MACHP, MAXT, N, AND NM ARE UNCHANGED.
C
C  WORKING VARIABLES:   XA,XB,DA,DB,EA,EB,FA,FB,SD,SE,G,H
C
      INTEGER NM,N,MAXT,ERR,I,J,K,L,M,LT,LM,PN,JA,G(NM),H(NM),P(NM),NN
      REAL A(NM,NM),B(NM,NM),C(NM,NM),XA(NM,NM),XB(NM,NM),DA(NM),DB(NM),
     +     EA(NM),EB(NM),FA(NM),FB(NM),SC(NM),SD(NM),SE(NM),MACHP,MACHPD
     +     ,EIGVAL,LB,UB,RD,FN
C
C  FN IS THE FUNCTION FOR X-1 IN THE 1,X FORM OF THE ADJACENCY MATRIX.
C
      FN=N/2
      NN=N-1
      PN=1000
C
C  SEARCH FOR AN OPTIMAL EIGENVECTOR AMONG THOSE THAT CORRESPOND TO AN
C  EIGENVALUE OF MULTIPLICITY ONE.
C
      DO 90 I=1,N
C
C  UNIQUELY MARK THE I TH NODE OF A AND THE NODES CONNECTED TO IT.
C
         DO 10 J=1,N
            DA(J)=A(I,J)
            A(J,J)=A(J,J)+DA(J)
            A(I,J)=3*FN
```

```
            A(J,I)=3*FN
   10       CONTINUE
            A(I,I)=A(I,I)+FN
C
C  COMPUTE THE EIGENVALUES AND EIGENVECTORS OF THIS SUBGRAPH.  A IS NOT
C  DESTROYED.
C
            CALL TRED2 (NM,N,A,DB,FB,XA)
            CALL TQL2 (NM,N,DB,FB,XA,ERR)
            ERR=-ERR
            IF (ERR.NE.0) RETURN
C
C  RESTORE A TO ITS ORIGINAL VALUE.
C
            DO 20 J=1,N
               A(J,J)=A(J,J)-DA(J)
               A(I,J)=DA(J)
               A(J,I)=DA(J)
   20       CONTINUE
C
C  SEARCH FOR THE OPTIMAL EIGENVECTOR OF THOSE JUST COMPUTED.
C
            DO 80 J=1,N
               LT=SC(I)
               MACHPD=MACHP*(1+ABS(DB(J)))
C
C  CHECK THAT THE ASSOCIATED EIGENVALUE HAS MULTIPLICITY ONE.
C
               IF (J.EQ.1) GO TO 30
                  IF (ABS(DB(J)-DB(J-1)).LT.MACHPD) GO TO 80
                     IF (J.EQ.N) GO TO 40
   30          IF (ABS(DB(J)-DB(J+1)).LT.MACHPD) GO TO 80
C
C  FIND AND RECORD THE NUMBER OF DUPLICATED PAIRS OF COMPONENTS.  IF A
C  TRIPLICATE IS FOUND, DISCONTINUE CONSIDERATION OF THAT EIGENVECTOR.
C
   40             DO 60 L=1,NN
                     M=L+1
                     LM=0
                     DO 50 K=M,N
                        IF (ABS(XA(L,J)-XA(K,J)).GT.MACHP) GO TO 50
                           IF (LM.EQ.1) GO TO 80
                              LM=1
                              LT=LT+1
   50                CONTINUE
   60             CONTINUE
                  IF (LT.GE.PN) GO TO 80
C
C  UPDATE CURRENT OPTIMAL EIGENVECTOR INFORMATION.
C
                     PN=LT
                     JA=I
                     EIGVAL=DB(J)
                     DO 70 K=1,N
                        SD(K)=XA(K,J)
   70                CONTINUE
C
C  IF THE EIGENVECTOR IS SUITABLE  -NOT NECESSARILY OPTIMAL-  TERMINATE
```

```
C  THE SEARCH.
C
                    IF (LT.LT.10) GO TO 100
   80      CONTINUE
   90   CONTINUE
C
C  IF NO SUITABLE EIGENVECTOR COULD BE FOUND, TERMINATE THIS PORTION
C  OF THE ALGORITHM WITH APPROPRIATE MESSAGE.
C
        IF (PN.LT.1000) GO TO 100
           ERR=3
           RETURN
  100   MACHPD=MACHP*(1+ABS(EIGVAL))
        LB=EIGVAL-MACHPD
        UB=EIGVAL+MACHPD
C
C  STORE THE SELECTED EIGENVECTOR IN THE FIRST COLUMN OF XA.
C
        DO 110 I=1,N
           XA(I, 1)=SD(I)
  110   CONTINUE
C
C  SELECTIVELY STORE THE PERTINENT INFORMATION HELD IN C AND SC IN ORDER
C  TO FREE STORAGE FOR FUTURE USE.
C
        M=SC(JA)
        DO 120 I=1,M
           G(I)=C(JA,I)
  120   CONTINUE
C
C  FIND AND EXAMINE THE CORRESPONDING EIGENVECTOR IN THE SUBGRAPHS OF B
C  WHICH HAVE SPECTRA MATCHING THE SPECTRA GENERATED FROM THE SUBGRAPH
C  OF A THAT PRODUCED THE OPTIMAL EIGENVECTOR.
C
        DO 170 L=1,M
C
C  COPY B INTO C.
C
           DO 140 I=1,N
              DO 130 J=1,N
                 C(I,J)=B(I,J)
  130         CONTINUE
  140      CONTINUE
C
C  FIND AND UNIQUELY MARK THE APPROPRIATE NODE IN B.
C
           I=G(L)
           DO 150 J=1,N
              C(J,J)=C(J,J)+C(I,J)
              C(I,J)=3*FN
              C(J,I)=3*FN
  150      CONTINUE
           C(I,I)=C(I,I)+FN
C
C  FIND THE DESIRED EIGENVECTOR  -  C IS DESTROYED.
C
           CALL TRED1 (NM,N,C,DA,DB,SC)
           CALL BISECT (N,0,DA,DB,SC,LB,UB,1,PN,RD,LM,ERR,SD,SE)
```

```
         ERR=-2*ERR-PN+1
         IF (ERR.NE.0) RETURN
         CALL TINVIT (NM,N,DA,DB,SC,PN,RD,LM,FB,ERR,SD,SE,EA,EB,FA)
         ERR=-ERR
         IF (ERR.NE.0) RETURN
         CALL TRBAK1 (NM,N,C,DB,PN,FB)
C
C  STORE THE EIGENVECTOR IN THE FIRST COLUMN OF XB.
C
         DO 160 J=1,N
            XB(J, 1)=FB(J)
 160     CONTINUE
C
C  SINCE BOTH EIGENVECTORS ARE STORED IN COLUMN ONE OF MATRIX ARRAYS,
C  SUBROUTINE SINGLE CAN BE USED TO GENERATE AND TEST UP TO MAXT
C  PERMUTATIONS.
C
         CALL SINGLE (A,B,XA,XB,DA,DB,H,P,ERR,MACHP,MAXT, 1,N,NM)
         IF (ERR.EQ.0) RETURN
         IF (ERR.EQ.4) NN=N
 170  CONTINUE
      ERR=1
      IF (NN.EQ.N) ERR=2
C
      RETURN
      END
```

4. Computational Experience

The UNIVAC 1108 computer at the National Bureau of Standards was used to test the computer program on well over 200 pairs of graphs specifically selected for use because of their difficulty. The results have been encouraging. A Brookhaven National Laboratories group has compiled [2] a list of 149 pairs of nonisomorphic graphs of order varying from 7 to 10 that have 0,1 adjacency matrices with identical spectra. These cospectral graphs arise in connection with the energy field of a Heisenberg model ferromagnet. The computer program previously listed distinguished each of the 149 pairs in a total time of 3.9 cpu seconds or, equivalently, each pair of graphs was determined to be nonisomorphic in approximately 0.026 cpu seconds. Of the graphs, 109 failed the valence test, 28 the node duplication test, and the remaining 12 pairs failed to have $1,x$ modified adjacency matrices with identical spectra.

With some modification the algorithm can be programmed to sort a large set of graphs into subsets characterized by node valences, node duplication, $1,x$ modified adjacency matrix spectra, and in more difficult cases, submatrix spectra. Once this is accomplished, the groups in each subset can be pairwise checked for isomorphism using the program listed in this paper. This procedure was employed on a collection of 35 graphs ranging in order from 5 to 9. Many of the graphs selected were known to be isomorphic and all had large automorphism groups, thus making it as difficult as possible for the algorithm to find permutations or to determine that none exist. By means of a simplified form of the sorting procedure, the 35 graphs were divided into 19 subsets in approximately 5 cpu seconds. In this particular case, the spectra test was a sufficient condition for graph isomorphism. To be sure, however, permutations were generated between each pair of graphs in each subset. A total of 16 permutations were generated, 13 by means of SUBROUTINE SINGLE and 3 requiring SUBROUTINE DOUBLE, in a total time of 1.6 cpu seconds.

None of the graphs yet tested, however, required the use of SUBROUTINE SUBVAL or SUBROUTINE SUBVEC. In response to a July, 1975 query about the existence of more challenging graphs, D. Corneil provided 16 pairwise nonisomorphic, order 25 graphs, any two of which required approximately 60 seconds on an IBM 370 (no model number was mentioned) to be distinguished by then current methods. The graphs were 3-strongly regular and stochastic with node valence 12. None of the graphs has duplicate nodes and each had identical $1,x$ adjacency matrix spectra. The submatrix spectra test, however, successfully sorted the group into 8 subsets, each containing two graphs. It is interesting to note that the two graphs contained in each subset were duals of each other. This initial sorting was accomplished in just 86 cpu seconds. When the algorithm was run on the 8 pairs of graphs, 5 were separated by means of the submatrix eigenvector test. The test was inconclusive on the remaining 3 pairs of graphs. The total run time was 150 cpu seconds. Of this total, 86 cpu seconds was spent distinguishing the 5 pairs and 64 cpu seconds on the unsuccessful effort to resolve the final 3 pairs. In addition, the algorithm was applied to 5 of the graphs and constructed permutations. The program produced a permutation relating each of the 5 pairs of graphs by means of SUBROUTINE SUBVEC in 81 cpu seconds.

Approximately 90 percent of the time required by the program is used in the calculation of eigenvalues and eigenvectors. Thus selection of optimal eigenvalue and eigenvector computing software is critical when time restrictions are sizeable. Since the EISPACK software referenced in the listing of the algorithm proved to be satisfactory for our needs, no significant effort was made to find more efficient software.

Tests on large numbers of random graphs were not conducted as it was felt that such tests would yield little meaningful data. Experience gained from working with the eigenvectors and eigenvalues of many graphs showed that only a relatively minute number of pairs of graphs exist for which the question of isomorphism is at all challenging for the algorithm and no such graphs are likely to occur in even a very large sample of randomly generated graphs. For the average pair of isomorphic graphs of order n, it is estimated that a permutation will be generated in well under $10^{-4}n^3$ cpu seconds UNIVAC 1108 time. Even faster times are achievable for random nonisomorphic pairs of graphs or through modifications in programming which first sort graphs by node valences, node duplicate structure $1,x$ modified adjacency matrix spectra, and in special cases (such as Corneil's collection of order 25 graphs) submatrix spectra, and then use the algorithm described in this paper to further reduce the subsets into classes for which the graphs are pairwise isomorphic.

Thus, with efficient application of the algorithm it is possible to divide a very large number of graphs into isomorphism equivalence classes in a reasonable amount of time. It is further possible to test pairs of very large graphs for isomorphism and if one exists, to generate permutations between two very large graphs.

5. References

[1.] Baker, G., Drum shapes and isospectral graphs, J. Math. Physics **7**, 2238–2242 (1966).

[2.] Baker, G., et al., A Data Compendium of Linear Graphs with Application to the Heisenberg Model, Brookhaven National Laboratory Report, 1967.

[3.] Corneil, D., and Gotlieb, C., An Efficient algorithm for graph isomorphism, JACM **17**, 51–64 (1970).

[4.] Corneil, D., The Analysis of Graph Theoretical Algorithms, Proc. Fifth S-E Conference on Combinatorics, Graph Theory and Computing, pp. 3–38 (1974).

[5.] Harary, F., King, C., Mowshowitz, A., and Read, R., Cospectral graphs and digraphs, Bull. London Math. Soc. **3**, 321–328 (1971).

[6.] Johnson, C., and Newman, M., A Note on Cospectral Graphs, submitted.

[7.] Mathon, R., and Corneil, D., Private Communications 1975–76.

[8.] Schmidt, D., and Druffel, L., A fast backtracking algorithm to test directed graphs for isomorphism using distance matrices, JACM **23**, 433–445 (1976).

[9.] Smith, B., et al., Matrix Eigensystem Routines — EISPACK Guide, Lecture Notes in Computer Science 6, (Springer-Verlag, New York, 1974).

(Paper 80B4–457)

JOURNAL OF RESEARCH of the National Bureau of Standards – B. Mathematical Sciences
Vol. 80B, No. 4, October-December 1976

Publications of the National Bureau of Standards*

Citations with Selected Abstracts

J. Res. Nat. Bur. Stand. (U.S.), **80A** *(Phys. and Chem.),* Nos. 5 and 6 Dec. 1976), SD Catalog No. C13.22/sec.A:80/5 and 6.

Deviation of international practical temperatures from thermodynamic temperatures in the temperature range from 273.16 K to 730 K, L. A. Guildner and R. E. Edsinger.

Measurements of the specific heats, C_σ, and C_v, of dense gaseous and liquid ethane, H. M. Roder.

Phase equilibria and crystal growth in the alkali antimonate systems Sb_2O_4-$NaSbO_3$, Sb_2O_4-$KSbO_3$, and Sb_2O_4-$NaSbO_3$-NaF, J. L. Waring, R. S. Roth, H. S. Parker, and W. S. Brower, Jr.

Vapor pressure formulation for water in range 0 to 100 °C. A revision, A. Wexler.

Revised lifetimes of energy levels in neutral iron, C. H. Corliss and J. L. Tech.

J. Res. Nat. Bur. Stand. (U.S.), **80B** *(Math. Sci.),* No. 3 (July-Sept. 1976), SD Catalog No. C13.22/sec.B:80/3.

A note on pairs of matrices with product zero, C. R. Johnson.

Finding the circuits of a matroid, E. Minieka.

One-sided tolerance limits for the normal distribution, $P = 0.80, \gamma = 0.80$, R. H. Wampler.

Spectral measures and separation of variables, D. W. Fox.

A survey of selected aspects of stratified and rotating fluids, R. G. Rehm.

A more tractable solution to a singular integral equation obtained by solving a related Hilbert problem for two unknowns, J. A. Pennline.

Sampling expansion for a Laguerre-L_r^α transform, A. J. Jerri.

Scaled equation of state parameters for gases in the critical region, J. M. H. L. Sengers, W. L. Greer, and J. V. Sengers, *J. Phys. Chem. Ref. Data* **5,** No. 1, 1-52 (1976).

Microwave spectra of molecules of astrophysical interest. IX. Acetaldehyde, A. Bauder, F. J. Lovas, and D. R. Johnson, *J. Phys. Chem. Ref. Data* **5,** No. 1, 53-78 (1976).

Microwave spectra of molecules of astrophysical interest. X. Isocyanic acid, G. Winnewisser, W. H. Hocking, and M. C. L. Gerry, *J. Phys. Chem. Ref. Data* **5,** No. 1, 79-102 (1976).

Diffusion in copper and copper alloys. Part IV. Diffusion in systems involving elements of group VIII, D. B. Butrymowicz, J. R. Manning, and M. E. Read, *J. Phys. Chem. Ref. Data* **5,** No. 1, 103-200 (1976).

A critical review of the Stark widths and shifts of spectral lines from non-hydrogenic atoms, N. Konjevic and J. R. Roberts, *J. Phys. Chem. Ref. Data* **5,** No. 2, 209-258 (1976).

Experimental Stark widths and shifts for non-hydrogenic spectral lines of ionized atoms. (A critical review and tabulation of selected data), N. Konjevic and W. L. Wiese, *J. Phys. Chem. Ref. Data* **5,** No. 2, 259-308 (1976).

Atlas of the absorption spectrum of nitric oxide (NO) between 1420 and 1250 Å, E. Miescher and F. Alberti, *J. Phys. Chem. Ref. Data* **5,** No. 2, 309-318 (1976).

Ideal gas thermodynamic properties of propanone and 2-butanone, J. Chao and B. J. Zwolinski, *J. Phys. Chem. Ref. Data* **5,** No. 2, 319-328 (1976).

Refractive index of alkali halides and its wavelength and temperature derivatives, H. H. Li, *J. Phys. Chem. Data* **5,** No. 2, 329-528 (1976).

Tables of critically evaluated oscillator strengths for the lithium isoelectronic sequence, G. A. Martin and W. L. Wiese, *J. Phys. Chem. Ref. Data* **5,** No. 3, 537-570 (1976).

Ideal gas thermodynamic properties of six chlorofluoromethanes, S. S. Chen, R. C. Wilhoit, and B. J. Zwolinski, *J. Phys. Chem. Ref. Data* **5,** No. 3, 571-580 (1976).

Survey of superconductive materials and critical evaluation of selected properties, B. W. Roberts, *J. Phys. Chem. Ref. Data* **5,** No. 3, 581-822 (1976).

Nuclear spins and moments, G. H. Fuller, *J. Phys. Chem. Ref. Data* **5,** No. 4, 835-1092 (1976).

Nuclear moments and moment ratios as determined by Mössbauer spectroscopy, J. G. Stevens and B. D. Dunlap, *J. Phys. Chem. Ref. Data* **5,** No. 4, 1093-1122 (1976).

Rate coefficients for ion-molecule reactions I. Ions containing C and H, L. W. Sieck and S. G. Lias, *J. Phys. Chem. Ref. Data* **5,** No. 4, 1123-1146 (1976).

Microwave spectra of molecules of astrophysical interest. XI. Silicon Sulfide, E. Tiemann, *J. Phys. Chem. Ref. Data* **5,** No. 4, 1147-1156 (1976).

Monogr. 25, Section 13. **Standard x-ray diffraction powder patterns. Section 13 – Data for 58 substances,** M. C. Morris, H. F. McMurdie, E. H. Evans, B. Paretzkin, J. H. de Groot, C. R. Hubbard, and S. J. Carmel, *Nat. Bur. Stand. (U.S.), Monogr. 25, Sec. 13,* 114 pages (June 1976) SD Catalog No. C13.44:25/Sec. 13, $1.80.

Monogr. 151. **Automatic measurement of network parameters – A survey,** R. W. Beatty, *Nat. Bur. Stand. (U.S.), Monogr. 151,* 113 pages (June 1976) SD Catalog No. C13.44:151, $4.50.

Monogr. 153. **The first spectrum of hafnium (Hf I),** W. F. Meggers and C. E. Moore, *Nat. Bur. Stand. (U.S.), Monogr. 153,* 117 pages (July 1976) SD Catalog No. C13.44:153, $1.35.

Monogr. 154. **The dynamics of fields of higher spin,** R. W. Hayward, *Nat. Bur. Stand. (U.S.), Monogr. 154,* 99 pages (Aug. 1976) SD Catalog No. C13.44:154, $1.70.

**Publications with prices and SD Catalog numbers may be purchased directly from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (foreign: one-fourth additional). Microfiche copies of the above, plus paper copy and microfiche versions of NBS Interagency Reports (NBSIR's), may be purchased from the National Technical Information Service (NTIS), Springfield, Va. 22161. Prices are subject to change. Reprints from the NBS Journal of Research and non-NBS publications may often be obtained directly from the authors.*

H119. **Quieting: A practical guide to noise control,** R. D. Berendt, E. L. R. Corliss, and M. S. Ojalvo, *Nat. Bur. Stand. (U.S.), Handb. 119,* 163 pages (July 1976) SD Catalog No. C13.11:119, $5.10.

SP305. Supplement 7. **Publications of the National Bureau of Standards 1975 catalog. A compilation of abstracts and key word and author indexes,** B. L. Hurdle, Ed., *Nat. Bur. Stand. (U.S.), Spec. Publ. 305 Suppl. 7,* 595 pages (June 1976) SD Catalog No. C13.10:305 Suppl. 7, $7.55.

SP384. **Annotated bibliography of the literature on resource sharing computer networks. Revised 1976,** H. M. Wood, S. W. Watkins, and I. W. Cotton, *Nat. Bur. Stand. (U.S.), Spec. Publ. 384 (revised),* 179 pages (Sept. 1976) SD Catalog No. C13.10:384/rev., $2.45.

SP396-4. **Critical surveys of data sources: Electrical and magnetic properties of metals,** M. J. Carr, R. B. Gavert, R. L. Moore, H. W. Wawrousek, and J. H. Westbrook, *Nat. Bur. Stand. (U.S.), Spec. Publ. 396-4,* 92 pages (July 1976) SD Catalog No. C13.10:396-4, $1.55.

SP400-22. **Semiconductor measurement technology: Microelectronic test pattern NBS-3 for evaluating the resistivity-dopant density relationship of silicon,** M. G. Buehler, *Nat. Bur. Stand. (U.S.), Spec. Publ. 400-22,* 57 pages (June 1976) SD Catalog No. C13.10:400-22, $1.45.

SP400-25. **Semiconductor measurement technology. Progress Report, July 1 to December 31, 1975,** W. M. Bullis, Ed., *Nat. Bur. Stand. (U.S.), Spec. Publ. 400-25,* 87 pages (Oct. 1976) SD Catalog No. C13.10:400-25, $1.45.

SP400-28. **Semiconductor measurement technology: NBS/FDA workshop. Reliability technology for cardiac pacemakers,** H. A. Schafft, Ed., *Nat. Bur. Stand. (U.S.), Spec. Publ. 400-28,* 49 pages (June 1976) SD Catalog No. C13.10:400-28, $1.25.

SP403. **Energy conservation through effective energy utilization.** 1973 Engineering Foundation Conference, New England College, Henniker, NH, August 19-24, 1973, J. C. Denton, S. Webber, and J. Moriarty, Eds., *Nat. Bur. Stand. (U.S.), Spec. Publ. 403,* 251 pages (June 1976) SD Catalog No. C13.10:403, $3.30.

SP422. Volume I and II. **Accuracy in trace analysis: Sampling, sample handling, analysis.** Proceedings of the 7th Materials Research Symposium held at the National Bureau of Standards, Gaithersburg, MD, October 7-11, 1974, P. D. LaFleur, Ed., *Nat. Bur. Stand. (U.S.), Spec. Pub. 422, Vol. I,* 645 pages, *Vol. II,* 636 pages, (Aug. 1976) SD Catalog No. C13.10:422/vol. I and II, $20 per 2-part set.

SP431. **Fourth international congress for stereology.** Proceedings of the Fourth International Congress for Stereology held at the National Bureau of Standards, Gaithersburg, Md., Sept. 4-9, 1975, E. E. Underwood, R. de Wit, and G. A. Moore, Eds., *Nat. Bur. Stand. (U.S.), Spec. Publ. 431,* 547 pages (Jan. 1976) SD Catalog No. C13.10:431, $11.00.

SP433. **MFPG—Success by design: Progress through failure analysis.** Proceedings of the 21st Meeting of the Mechanical Failures Prevention Group, held at the National Bureau of Standards, Gaithersburg, MD, November 7-8, 1974, T. R. Shives and W. A. Willard, Eds., *Nat. Bur. Stand. (U.S.), Spec. Publ. 433,* 229 pages (July 1976) SD Catalog No. C13.10:433, $3.50.

SP443. **Hydraulic research in the United States and Canada, 1974,** G. Kulin and P. H. Gurewitz, Eds., *Nat. Bur. Stand. (U.S.), Spec. Publ. 443,* 359 pages (June 1976) SD Catalog No. C13.10:443, $4.15.

SP445-1. **The National Measurement System for Time and Frequency,** A. S. Risley, *Nat. Bur. Stand. (U.S.), Spec. Publ. 445-1,* 72 pages (June 1976) SD Catalog No. C13.10:445-1, $1.20.

SP446. **Building technology project summaries,** M. Olmert, *Nat. Bur. Stand. (U.S.), Spec. Publ. 446,* 108 pages (May 1976) SD Catalog No. C13.10:446, $2.05.

SP448. **Automation technology applied to public service.** Proceedings of a Conference on Automation Technology Applied to Public Service, held at the National Bureau of Standards, Gaithersburg, MD, May 21-22, 1974, E. G. Johnsen, Ed., *Nat. Bur. Stand. (U.S.), Spec. Publ. 448,* 87 pages (Sept. 1976) SD Catalog No. C13.10:448, $1.90.

SP451. **Data base directions. The next steps.** Proceedings of the Workshop of the National Bureau of Standards and the Association for Computing Machinery, held at Fort Lauderdale, Florida, October 29-31, 1975, J. L. Berg, Ed., R. G. Canning, D. L. Adams, R. W. Bemer, G. Dodd, R. M. Gall, and C. D. Trigg, *Nat. Bur. Stand. (U.S.), Spec. Publ. 451,* 175 pages (Sept. 1976) SD Catalog No. C13.10:451, $2.40.

SP452. **MFPG—The role of coatings in the prevention of mechanical failures.** Proceedings of the 23d Meeting of the Mechanical Failures Prevention Group, held at the National Bureau of Standards, Gaithersburg, MD, Oct. 29-31, 1975, T. R. Shives and W. A. Willard, Eds., *Nat. Bur. Stand. (U.S.), Spec. Publ. 452,* 199 pages (Sept. 1976) SD Catalog No. C13.10:452, $2.65.

SP454. **An annotated bibliography of compiled thermodynamic data sources for biochemical and aqueous systems (1930 to 1975). Equilibrium, enthalpy, heat capacity, and entropy data,** G. T. Armstrong and R. N. Goldberg, *Nat. Bur. Stand. (U.S.), Spec. Publ. 454,* 67 pages (Sept. 1976) SD Catalog No. C13.10:454, $1.65.

NSRDS-NBS57. **Yields of free ions formed in liquids by radiation,** A. O. Allen, *Nat. Stand. Ref. Data Ser., Nat. Bur. Stand. (U.S.), 57,* 25 pages (Apr. 1976) SD Catalog No. C13.48:57, 55 cents.

NSRDS-NBS58. **Drift mobilities and conduction band energies of excess electrons in dielectric liquids,** A. O. Allen, *Nat. Stand. Ref. Data Ser., Nat. Bur. Stand. (U.S.), 58,* 23 pages (May 1976) SD Catalog No. C13.48:58, 70 cents.

BSS69. **NBSLD, the computer program for heating and cooling loads in buildings,** T. Kusuda, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 69,* 398 pages (July 1976) SD Catalog No. C13.29/2:69, $4.60.

BSS73. **Structural performance of masonry walls under compression and flexure,** S. G. Fattal and L. E. Cattaneo, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 73,* 65 pages (June 1976) SD Catalog No. C13.29/2:73, $1.25.

BSS76. **Analysis of reinforced concrete beams subjected to fire,** B. Ellingwood and J. R. Shaver, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 76,* 83 pages (July 1976) SD Catalog No. C13.29/2:76, $1.45.

BSS86. **Engineering aspects of cyclone Tracy, Darwin, Australia, 1974,** R. D. Marshall, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 86,* 39 pages (June 1976) SD Catalog No. C13.29/2:86, $1.05.

BSS87. **Model documents for the evaluation, approval, and inspection of manufactured buildings,** P. W. Cooke, R. D. Dikkers, H. R. Trechsel, H. K. Tejuja, and L. P. Zelenka, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 87,* 296 pages (July 1976) SD Catalog No. C13.29/2:87, $4.40.

BSS90. **The structure of building specifications,** S. J. Fenves, K. Rankin, and H. K. Tejuja, *Nat. Bur. Stand. (U.S.), Bldg. Sci. Ser. 90*, 83 pages (Sept. 1976) SD Catalog No. C13.29/2:90, $1.45.

FIPS PUB 40. **Guideline for optical character recognition forms,** T. C. Bagg, Standard Coordinator, *Nat. Bur. Stand. (U.S.), Fed. Info. Process. Stand. Publ. (FIPS PUB) 40*, 60 pages (1976) SD Catalog No. 13.52:40, $1.80.

PS66-75. **Safety requirements for home playground equipment.** *(ANS Z 304.1-1975),* J. M. Tascher, Technical Standards Coordinator, *Nat. Bur. Stand. (U.S.), Prod. Stand. 66-75*, 14 pages (July 1976) SD Catalog No. C13.20/2:66.75, 45 cents.

TN594-11. **Optical radiation measurements: Development of an NBS reference spectrophotometer for diffuse transmittance and reflectance,** W. H. Venable, Jr., J. J. Hsia, and V. R. Weidner, *Nat. Bur. Stand. (U.S.), Tech. Note 594-11*, 47 pages (Oct. 1976) SD Catalog No. C13.46:594-11, $1.15.

TN594-12. **Optical radiation measurements: The translucent blurring effect—Method of evaluation and estimation,** J. J. Hsia, *Nat. Bur. Stand. (U.S.), Tech. Note 594-12*, 31 pages (Oct. 1976) SD Catalog No. C13.46:594-12, 75 cents.

TN680. **Application of infrared frequency synthesis techniques with metal-insulator-metal diodes to the spin flip Raman laser,** J. S. Wells, G. E. Streit, and F. R. Petersen, *Nat. Bur. Stand. (U.S.), Tech. Note 680*, 14 pages (Apr. 1976) SD Catalog No. C13.46:680, 50 cents.

TN681. **A satellite-controlled digital clock,** J. V. Cateora, D. D. Davis, and D. W. Hanson, *Nat. Bur. Stand. (U.S.), Tech. Note 681*, 46 pages (June 1976) SD Catalog No. C13.46:681, $1.05.

TN682. **Implementation of the notch technique as an rf peak pulse power standard,** P. A. Simpson and P. A. Hudson, *Nat. Bur. Stand. (U.S.), Tech. Note 682*, 30 pages (July 1976) SD Catalog No. C13.46:682, 80 cents.

TN683. **Models for the interpretation of frequency stability measurements,** J. A. Barnes, *Nat. Bur. Stand. (U.S.), Tech. Note 683*, 44 pages (Aug. 1976) SD Catalog No. C13.46:683, $1.00.

TN906. **A methodology for evaluating alternative technical and information management approaches to privacy requirements,** R. C. Goldstein, H. H. Seward, and R. L. Nolan, *Nat. Bur. Stand. (U.S.), Tech. Note 906*, 72 pages (June 1976) SD Catalog No. C13.46:906, $1.35.

TN908. **Measurement of computer communication networks,** M. D. Abrams, S. Treu, and R. P. Blanc, *Nat. Bur. Stand. (U.S.), Tech. Note 908*, 95 pages (July 1976) SD Catalog No. C13.46:908, $1.55.

TN913. **Modification of an NBS reference spectrophotometer,** K. L. Eckerle, *Nat. Bur. Stand. (U.S.), Tech. Note 913*, 43 pages (July 1976) SD Catalog No. C13.46:913, $1.00.

TN914. **A new dynamic pressure source for the calibration of pressure transducers,** C. F. Vezzetti, J. S. Hilten, J. F. Mayo-Wells, and P. S. Lederer, *Nat. Bur. Stand. (U.S.), Tech. Note 914*, 35 pages (June 1976) SD Catalog No. C13.46:914, 85 cents.

TN917. **A review of network access techniques with a case study: The network access machine,** R. Rosenthal, *Nat. Bur. Stand. (U.S.), Tech. Note 917*, 36 pages (July 1976) SD Catalog No. C13.46:917, 85 cents.

TN918. **Survey of building code provisions for historic structures,** M. Green and P. W. Cooke, *Nat. Bur. Stand. (U.S.), Tech. Note 918*, 47 pages (Sept. 1976) SD Catalog No. C13.46:918, $1.15.

TN919. **Operating system structures to support security and reliable software,** T. A. Linden, *Nat. Bur. Stand. (U.S.), Tech. Note 919*, 51 pages (Aug. 1976) SD Catalog No. C13.46:919, $1.25.

TN920. **Use of synthetic benchmarks for estimating service bureau processing charges,** D. M. Conti, *Nat. Bur. Stand. (U.S.), Tech. Note 920*, 47 pages (July 1976) SD Catalog No C13.46:920, $1.15.

TN921. **Survey manual for estimating the incidence of lead paint in housing,** W. G. Hall and L. T. Slovic, *Nat. Bur. Stand. (U.S.), Tech. Note 921*, 111 pages (Sept. 1976) SD Catalog No. C13.46:921, $2.20.

TN922. **Economic analysis of experimental lead paint abatement methods: Phase I,** R. E. Chapman, *Nat. Bur. Stand. (U.S.), Tech. Note 922*, 112 pages (Sept. 1976) SD Catalog No. C13.46:922, $1.75.

TN923. **Applications of thermography for energy conservation in industry,** C. W. Hurley and K. G. Kreider, *Nat. Bur. Stand. (U.S.), Tech. Note 923*, 31 pages (Oct. 1976) SD Catalog No. C13.46:923, 75 cents.

TN924. **Procedures for the calibration of ASTM E127-type ultrasonic reference blocks,** D. J. Chwirut, G. F. Sushinsky, and D. G. Eitzen, *Nat. Bur. Stand. (U.S.), Tech. Note 924*, 43 pages (Sept. 1976) SD Catalog No. C13.46:924, $1.05.

TN925. **Overviews and justifications for low gravity experiments on phase transition and critical phenomena in fluids,** M. R. Moldover, R. J. Hocken, R. W. Gammon, and J. V. Sengers, *Nat. Bur. Stand. (U.S.), Tech. Note 925*, 99 pages (Oct. 1976) SD Catalog No. C13.46:925, $1.70.

TN930. **A measurement assurance program for electric energy,** N. M. Oldham, *Nat. Bur. Stand. (U.S.), Tech. Note 930*, 23 pages (Sept. 1976) SD Catalog No. C13.46:930, 55 cents.

NBSIR 75-735. **An expandable total energy data editor,** R. H. F. Jackson, 151 pages (Aug. 1976). Order from NTIS as PB256295, $6.00.

NBSIR 75-926. **The National Measurement System for length and related dimensional measurements, Part I,** J. W. Lazar and R. L. Bach, 115 pages (Aug. 1976). Order from NTIS as PB257467, $5.50.

NBSIR 75-930. **The national measurement system for fluid flow,** W. C. Haight, P. S. Klebanoff, F. W. Ruegg, and G. Kulin, 66 pages (Aug. 1976). Order from NTIS as PB258250, $4.50.

NBSIR 75-932. **The National Measurement System for temperature,** J. F. Schooley, 61 pages (Aug. 1976). Order from NTIS as PB257425, $4.50.

NBSIR 75-942. **The national measurement system for optics,** D. A. Swyt, 63 pages (Aug. 1976). Order from NTIS as PB256644, $4.50.

NBSIR 75-953. **A procedure for determining bicycle braking performance,** D. E. Marlowe, 26 pages (July 1976). Order from NTIS as PB256219, $4.00.

NBSIR 75-961. **The police patrol car: Economic efficiency in acquisition, operation, and disposition,** R. T. Ruegg, 225 pages (June 1976). Order from NTIS as PB257466, $7.75.

NBSIR 76-841. **An analytical and experimental determination of the cutoff frequencies of higher-order TE modes in a TEM cell,** J. C. Tippet, D. C. Chang, and M. L. Crawford, 27 pages (June 1976). Order from NTIS as PB256319, $4.00.

NBSIR 76-988. **Regional seminar on a system of standardization and metrology for Latin America,** H. S. Peiser and R. S. Marvin, Eds., 100 pages (Feb. 1976). Order from NTIS as PB254469, $5.00.

NBSIR 76-993. **The calibration of a pentaprism,** C. P. Reeve and R. C. Veale, 33 pages (Jan. 1976). Order from NTIS as PB253227, $4.00.

NBSIR 76-997. **Proceedings of procurement practices symposium—Health care, October 14-16, 1975,** J. G. Berke, Ed., 82 pages (Feb. 1976). Order from NTIS as PB254298, $5.00.

NBSIR 76-1002. **Industrial process data for fluids: A survey of current research at the National Bureau of Standards,** H. J. White, Jr., Editor, 55 pages (Apr. 1976). Order from NTIS as PB257469, $4.50.

NBSIR 76-1012. **Fire performance testing of bulkhead insulation systems for high strength to weight ship structures,** B. T. Lee, 64 pages (Aug. 1976). Order from NTIS as PB257197, $4.50.

NBSIR 76-1013. **Flame spread of carpet systems involved in room fires,** K. M. Tu and S. Davis, 42 pages (June 1976). Order from NTIS as PB256130, $4.00.

NBSIR 76-1016. **Mobile home smoke detector siting study,** W. M. Gawin and R. G. Bright, 55 pages (May 1976). Order from NTIS as PB254177, $4.50.

NBSIR 76-1018. **Evaluation of the fire performance of carpet underlayments,** S. Alderson and L. Breden, 69 pages (Sept. 1976). Order from NTIS as PB257779, $4.50.

NBSIR 76-1021. **Fire spread along a mobile home corridor,** E. K. Budnick, 58 pages (July 1976). Order from NTIS as PB257101, $4.50.

NBSIR 76-1023. **Direct comparisons of the NBS absorbed dose calorimeters irradiated with 20 and 50 MeV electrons,** S. R. Domen, P. J. Lamperti, and T. Mikado, 23 pages (July 1976). Order from NTIS as PB256191, $3.50.

NBSIR 76-1024. **Statistical analysis of blood lead levels of children surveyed in Pittsburgh, Pennsylvania: Analytical methodology and summary results,** W. D. Urban, 75 pages (Apr. 1976). Order from NTIS as PB255876, $4.50.

NBSIR 76-1031. **Shape dependence of light-scattering behavior of dust particles,** R. Zerull, 23 pages (June 1976). Order from NTIS as PB253932, $3.50.

NBSIR 76-1034. **Chemical thermodynamic properties of compounds of sodium, potassium and rubidium: An interim tabulation of selected values,** D. D. Wagman, W. H. Evans, V. B. Parker, and R. H. Schumm, 76 pages (Apr. 1976). Order from NTIS as PB254460, $5.00.

NBSIR 76-1037. **Integral measurement procedures for determining particle size in aerosols,** F. J. Kraus, 38 pages (May 1976). Order from NTIS as PB253933, $4.00.

NBSIR 76-1039. **Evaluation of selected connectors for aluminum wire in residential structures,** O. B. Laug, 37 pages (Mar. 1976). Order from NTIS as PB254047, $4.00.

NBSIR 76-1040. **Points of view in testing flame detectors,** M. Schnell, 24 pages (May 1976). Order from NTIS as PB254178, $3.50.

NBSIR 76-1041. **Security analysis and enhancements of computer operating systems,** The RISOS Project, Lawrence Livermore Laboratory, 70 pages (Apr. 1976). Order from NTIS as PB257087, $4.50.

NBSIR 76-1043. **An economic analysis of residential abandonment and rehabilitation,** P. F. Colwell, 111 pages (May 1976). Order from NTIS as PB254347, $5.50.

NBSIR 76-1046. **Improved building design through the psychology of perception: Perceptual selectivity applied to livability and safety with sample performance requirements,** N. Starnes, R. Wehrli, and R. Cormack, 103 pages (July 1976). Order from NTIS as PB256476, $5.50.

NBSIR 76-1052. **Naval shipboard fire risk criteria—Berthing compartment fire study and fire performance guidelines,** B. T. Lee and W. J. Parker, 86 pages (Sept. 1976). Order from NTIS as PB258118, $5.00.

NBSIR 76-1056. **Analysis of solar energy system for the GSA demonstration office building at Manchester, New Hampshire,** T. Kusuda, S. T. Liu, J. W. Bean, and J. P. Barnett, 35 pages (Mar. 1, 1976). Order from NTIS as PB254179, $4.00.

NBSIR 76-1059. **Intermediate minimum property standards for solar heating and domestic hot water systems,** Solar Energy Program Team, Center for Building Technology, 171 pages (Apr. 1976). Order from NTIS as PB257086, $6.75.

NBSIR 76-1060. **NBS interagency transducer project. Progress Report No. 4,** P. S. Lederer, J. S. Hilten, C. F. Vezzetti, and J. F. Mayo-Wells, 26 pages (June 10, 1976). Order from NTIS as PB253934, $4.00.

NBSIR 76-1061. **An annotated bibliography on proton affinities,** K. Hartman, S. Lias, P. J. Ausloos, and H. M. Rosenstock, 52 pages (July 1976). Order from NTIS as PB256328, $4.50.

NBSIR 76-1063. **Air leakage measurements in a mobile home,** C. M. Hunt, S. J. Treado, and B. A. Peavy, 27 pages (Aug. 1976). Order from NTIS as PB257102, $4.00.

NBSIR 76-1064. **Existing architectural information indexing systems,** R. J. Kapsch, 68 pages (Mar. 1976). Order from NTIS as PB254181, $4.50.

NBSIR 76-1066. **Center for consumer product technology publications, November 1974 to April 1976,** S. Halpin, 13 pages (June 1976). Order from NTIS as PB254473, $3.50.

NBSIR 76-1067. **Mathematical approaches to evaluating aircraft vertical separation standards,** J. F. Gilsinn and D. R. Shier, 50 pages (May 1976). Order from NTIS as PB257195, $4.00.

NBSIR 76-1069. **Environmental effects on the strength of a glass fiber-reinforced-plastic rod material,** N. Halsey and L. Mordfin, 38 pages (July 1976). Order from NTIS as PB256296, $4.00.

NBSIR 76-1072. **Back-up report for the proposed standard for the flammability of general wearing apparel,** E. Braun, V. B. Cobble, S. Helzer, J. F. Krasny, R. D. Peacock, and A. K. Stratton, 51 pages (June 1976). Order from NTIS as PB255446, $4.50.

NBSIR 76-1076. **Flammability limits: Thermodynamics and kinetics,** A. Maček, 27 pages (May 1976). Order from NTIS as PB254180, $4.00.

NBSIR 76-1081. **Progress report on the corrosion behavior of selected stainless steels in soil environments,** W. F. Gerhold, E. Escalante, and B. T. Sanderson, 77 pages (Aug. 1976). Order from NTIS as PB256329, $5.00.

NBSIR 76-1083. **Report to AID on a Philippines survey on standardization and measurement services,** H. S. Peiser and R. S. Marvin, Eds., 124 pages (June 1976). Order from NTIS as PB255803, $5.50.

NBSIR 76-1084. **Nondestructive examination of glass-reinforced-plastic rod end fittings,** G. F. Sushinsky and L. Mordfin, 28 pages (Aug. 1976). Order from NTIS as PB257180, $4.00.

NBSIR 76-1087. **On a scattered-light measuring device for use in testing types of smoke detectors,** M. Pistor, 35 pages (July 1976). Order from NTIS as PB257202, $4.00.

NBSIR 76-1089. **An analysis of inertial seismometer-galvanometer combinations,** D. P. Johnson and H. Matheson, 205 pages (June 1976). Order from NTIS as PB254475, $7.75.

NBSIR 76-1090. **Environmental effects on microphones of various constructions,** G. R. Hruska, E. B. Magrab, W. B. Penzes, 36 pages (July 1976). Order from NTIS as PB255505, $4.00

NBSIR 76-1091. **Materials for fuel cells,** L. H. Bennett, M. I. Cohen, A. L. Dragoo, A. D. Franklin, and A. J. McAlister, 39 pages (May 1976). Order from NTIS as PB257768, $4.00.

NBSIR 76-1093. **The application of test structures and test patterns to the development of radiation hardened integrated circuits: A review,** K. F. Galloway and M. G. Buehler, 15 pages (Aug. 1976). Order from NTIS as PB256318, $3.50.

NBSIR 76-1095. **Properties and interactions of oral structures and restorative materials,** J. M. Cassell, 77 pages (Aug. 1976). Order from NTIS as PB255808, $5.00.

NBSIR 76-1096. **Guidelines for the determination and development of generic safety standards,** R. G. Hendrickson, 58 pages (Aug. 1976). Order from NTIS as PB257076, $4.50.

NBSIR 76-1097. **Illustrative generic standard for the control of thermal burn hazards in household appliances,** R. G. Hendrickson, E. M. Robertson, and R. V. Kelly, 49 pages (Aug. 1976). Order from NTIS as PB257141, $4.00.

NBSIR 76-1099. **Standardization and measurement services in Bolivia,** H. S. Peiser, R. S. Marvin, and J. Mejeur, Eds., 105 pages (Sept. 1976). Order from NTIS as PB258612, $5.50.

NBSIR 76-1100. **Appraisal of Federal Government Cobol standards and software management: Survey results,** D. R. Deutsch, 75 pages (June 1976). Order from NTIS as PB258371, $4.50.

NBSIR 76-1102. **Computer programs for structural chemistry: STATUS. A FORTRAN program for statistical analysis of crystallographic quantities,** L. W. Schroeder and B. Dickens, 152 pages (July 1976). Order from NTIS as PB257729, $6.75.

NBSIR 76-1107. **Considerations in developing test methods for protective headgear,** R. E. Berger, 55 pages (Aug. 1976). Order from NTIS as PB258322, $4.50.

NBSIR 76-1109. **Architectural glazing safety standard: Survey of codes and standards,** S. C. Adler, 65 pages (Aug. 1976). Order from NTIS as PB257194, $4.50.

NBSIR 76-1110. **The acoustic pressure field alongside a manikin's head with a view towards *in situ* hearing aid tests,** E. D. Burnett and G. F. Kuhn, 26 pages (Aug. 1976). Order from NTIS as PB257073, $4.00.

NBSIR 76-1115. **Optical materials characterization,** A. Feldman, D. Horowitz, R. M. Waxler, and M. J. Dodge, 20 pages (Aug. 1976). Order from NTIS as PB257196, $3.50.

NBSIR 76-1120. **Fire research publications, 1975,** N. H. Jason, 17 pages (Sept. 1976). Order from NTIS as PB257837, $3.50.

NBSIR 76-1130. **Program and abstracts. Symposium on nonbiological transport and transformation of pollutants on land and water: Processes and critical data required for predictive description, May 11-13, 1976, National Bureau of Standards, Gaithersburg, MD,** L. H. Gevantman, Ed., 199 pages (May 1976). Order from NTIS as PB257347, $7.50.

NBSIR 76-1137. **Thermal data requirements and performance evaluation procedures for the national solar heating and cooling demonstration program,** E. Streed, M. McCabe, D. Waksman, J. Hebrank, and T. Richtmyer, 86 pages (Aug. 1976). Order from NTIS as PB257770, $5.00.

NBSIR 76-1143. **Plan for the development and implementation of standards for solar heating and cooling applications,** D. Waksman, J. H. Pielert, R. D. Dikkers, and E. R. Streed, 55 pages (Aug. 1976). Order from NTIS as PB257769, $4.00.

This column lists all outside publications by the NBS staff, as soon as possible after issuance as practical. For completeness, earlier references not previously reported may be included from time to time.

15918. Stillman, R. B., **FORTRAN analysis by simple transforms,** *Proc. Computer Science and Statistics East Annual Symposium on the Interface, Los Angeles, Calif., Feb. 13-14, 1975,* 7 pages (1975).

15919. Unassigned.

15920. Kaufman, V., Sugar, J., **Resonance transition array of Yb IV,** *J. Opt. Soc. Am.* **66,** No. 5, 439 (May 1976).

15921. Joy, D. C., Leamy, H. J., Ferris, S. D., Yakowitz, H., Newbury, D. E., **Domain wall image contrast in the SEM,** *Appl. Phys. Lett.* **28,** No. 8, 466-467 (Apr. 15, 1976).

15922. Bower, V. E., Davis, R. S., **A new determination of the Faraday by means of the silver coulometer,** (Proc. 5th Int. Conf. on Atomic Masses and Fundamental Constants, Paris, France, June 1975), Paper in *Atomic Masses and Fundamental Constants* **5,** J. H. Sanders and A. H. Wapstra, Eds., Pt. 12, 578-583 (Plenum Press, New York, NY, 1976).

15923. Davis, R. M., **Managing computers for public service,** *Gov. Data Syst.*, pp. 17, 19 (July-Aug. 1973).

15924. Spector, N., Sugar, J., **Analysis of the fourth spectrum of terbium (Tb IV),** *J. Opt. Soc. Am.* **66,** No. 5, 436-438 (May 1976).

15925. Maki, A. G., Sams, R. L., **Infrared spectrum of $H^{12}C^{14}N$ and $H^{12}C^{15}N$,** *J. Mol. Spectrosc.* **60,** 57-62 (1976).

15926. Juvet, R. S., Jr., Cram, S. P., **Gas chromatography,** *Anal. Chem.* **46,** No. 5, 101R-124R (Apr. 1974).

15927. Julienne, P. S., Davis, J., **Cascade and radiation trapping effects on atmospheric atomic oxygen emission excited by electron impact,** *J. Geophys. Res.* **81,** No. 7, 1397-1403 (Mar. 1, 1976).

15928. Feldman, A., McKean, W. J., **Improved stressing apparatus for photoelasticity measurements,** *Rev. Sci. Instrum.* **46,** No. 11, 1588-1589 (Nov. 1975).

15929. Hubbard, C. R., Evans, E. H., Smith, D. K., **The reference intensity ratio, I/I_c, for computer simulated powder patterns,** *J. Appl. Crystallogr.* **9,** Part 2, 169-174 (Apr. 1976).

15930. Rosasco, G. J., Roedder, E., Simmons, J. H., **Laser-excited Raman spectroscopy for nondestructive partial analysis of**

individual phases in fluid inclusions in minerals, *Science* **190,** 557-560 (Nov. 7, 1975).

15931. Feldman, A., Waxler, R. M., Horowitz, D., **Measuring photoelastic and elastic constants of transparent materials by application of static stress,** (Proc. Int. Conf. on Optical Properties of Highly Transparent Solids, Waterville Valley, NH, Feb. 3-5, 1975), Paper in *Optical Properties of Highly Transparent Solids*, S. S. Mitra and B. Bendow, Eds., pp. 517-525 (Plenum Publishing Corp., New York, NY, 1975).

15932. Dewey, H. J., **Second-harmonic generation in $KB_5O_8{\cdot}4H_2O$ from 217.1 to 315.0 nm,** *IEEE J. Quantum Electron.* **QE-12,** No. 5, 303-306 (May 1976).

15933. Noyce, J. R., Hutchinson, J. M. R., Mann, W. B., Mullen, P. A., **Development of a National Bureau of Standards environmental radioactivity standard: River sediment,** (Proc. Int. Conf. on Environmental Sensing and Assessment, Las Vegas, Nev., Sept. 14-19, 1975), Paper in *International Conference on Environmental Sensing and Assessment*, **1,** Session 19-5, 1-9 (Institute of Electrical and Electronics Engineers, Inc., New York, NY, 1976).

15934. Olsen, P. T., Williams, E. R., **Determination of the gyromagnetic ratio of the proton γ_p',** (Proc. 5th Int. Conf. on Atomic Masses and Fundamental Constants, Paris, France, June 1975), Paper in *Atomic Masses and Fundamental Constants* **5,** J. H. Sanders and A. H. Wapstra, Eds., Part 11, 538-544 (Plenum Press, New York, NY, 1976).

15935. Bennett, H. S., Forman, R. A., **An alternative way to determine absorption coefficients in highly transparent solids: Theory,** (Proc. Int. Conf. on Optical Properties of Highly Transparent Solids, Waterville Valley, NH, Feb. 3-5, 1975), Paper in *Optical Properties of Highly Transparent Solids*, S. S. Mitra and B. Bendow, Eds., pp. 451-459 (Plenum Publishing Corp., New York, NY, 1975).

15936. Bennett, H. S., Forman, R. A., **Measuring absorption coefficients of highly transparent solids by photoacoustic methods: Cylindrical configurations,** (Proc. 5th Conf. on Infrared Laser Window Materials, Las Vegas, Nev., Dec. 1-4, 1975), Paper in *Proceedings of the 5th Conference on Infrared Laser Window Materials*, pp. 628-637 (Air Force Materials Laboratory, Wright-Patterson Air Force Base, OH, Feb. 1976).

15937. Feldman, A., Horowitz, D., Waxler, R. M., **Piezo-optical constants in the infrared,** (Proc. 5th Conf. on Infrared Laser Window Materials, Las Vegas, Nev., Dec. 1-4, 1975), Paper in *Proceedings of the 5th Conference on Infrared Laser Window Materials*, pp. 944-951 (Air Force Materials Laboratory, Wright-Patterson Air Force Base, OH, Feb. 1976).

15938. Feldman, A., Malitson, I. H., Horowitz, D., Waxler, R. M., Dodge, M. J., **Optical properties of polycrystalline zinc selenide,** (Proc. 4th Annual Conf. on Infrared Laser Window Materials, Tucson, Ariz., Nov. 18-20, 1974), Paper in *Proceedings of the 4th Annual Conference on Infrared Laser Window Materials*, pp. 118-129 (Air Force Materials Laboratory, Wright-Patterson Air Force Base, OH, Jan. 1975).

15939. Antonucci, J. M., Bowen, R. L., **Dimethacrylates derived from hydroxybenzoic acids,** *J. Dent. Res.* **55,** No. 1, 8-15 (1976).

15940. Gilsinn, D. E., **The method of averaging and domains of stability for integral manifolds,** *SIAM J. Appl. Math.* **29,** No. 4, 628-660 (Dec. 1975).

15941. Johnson, P. M., **The multiphoton ionization spectrum of benzene,** *J. Chem. Phys.* **64,** No. 10, 4143-4148 (May 15, 1976).

15942. Dunn, G. H., **Electron impact ionization,** Proc. Int. Conf. on Heavy Ion Sources, Gatlinburg, TN, Oct. 27-30, 1975, *IEEE Trans. Nucl. Sci.* **NS-23,** No. 2, 929-933 (Apr. 1976).

15943. Frederikse, H. P. R., Hosler, W. R., Armstrong, A. J., Negas, T., **Spinels for MHD-electrodes,** (Proc. 15th Symp. on Engineering Aspects of Magnetohydrodynamics, The University of Pennsylvania, Philadelphia, PA, May 24-26, 1976), Paper in *Proceedings of the 15th Symposium on Engineering Aspects of Magnetohydrodynamics, Session II.2*, 1-4 (Dept. of Mechanical Engineering, University of Mississippi, University, MS, 1976).

15944. Wasik, S. P., Brown, R. L., Minor, J. L., Jr., **Partition coefficients and solubility measurements of dimethylmercury in fresh and sea water over a temperature range 0-25 °C,** *J. Environ. Sci. Health—Environ. Sci. Eng.* **A11,** No. 1, 99-105 (1976).

15945. Hubbard, C. R., Mauer, F. A., **Precision and accuracy of the Bond method as applied to small spherical crystals,** *J. Appl. Cryst.* **9,** Part 1, 1-8 (Feb. 1976).

15946. Williams, J. G., Dicke, R. H., Bender, P. L., Alley, C. O., Carter, W. E., Currie, D. G., Eckhardt, D. H., Faller, J. E., Kaula, W. M., Mulholland, J. D., Plotkin, H. H., Poultney, S. K., Shelus, P. J., Silverberg, E. C., Sinclair, W. S., Slade, M. A., Wilkinson, D. T., **New test of the equivalence principle from lunar laser ranging,** *Phys. Rev. Lett.* **36,** No. 11, 551-554 (Mar. 15, 1976).

15947. Taylor, B. N., Cohen, E. R., **Present status of the fundamental constants,** (Proc. 5th Int. Conf. on Atomic Masses and Fundamental Constants, Paris, France, June 1975), Paper in *Atomic Masses and Fundamental Constants* **5,** J. H. Sanders and A. H. Wapstra, Eds., Part 14, 663-673 (Plenum Press, New York, NY, 1976).

15948. Mann, W. B., Schima, F. J., Unterweger, M. P., **Radioactivity standards of the Noble gases,** (Proc. Symp. on Noble Gases, Las Vegas, NV, Sept. 24-28, 1973), Chapter IV in *Noble Gases*, R. E. Stanley and A. A. Moghissi, Eds., ERDA TIC as Conf. 730915, pp. 144-153 (1975).

15949. Kurylo, M. J., Braun, W., **Flash photolysis resonance fluorescence study of the reaction $Cl+O_3 \rightarrow ClO+O_2$ over the temperature range 213-298 K,** *Chem. Phys. Lett.* **37,** No. 2, 232-235 (Jan. 15, 1976).

15950. Caswell, R. S., Coyne, J. J., **Microdosimetric spectra and parameters of fast neutrons,** *Proc. 5th Symp. on Microdosimetry, Verbania Pallanza, Italy, Sept. 22-26, 1975*, **EUR 5452** d-e-f, pp. 97-126 (1976).

15951. Brandt, M. A., Truhlar, D. G., **Test of the infinite-order sudden approximation for electron scattering at intermediate energy,** *Chem. Phys.* **13,** 461-467 (1976).

15952. Cornell, D., Tsang, W., **Thermal decomposition of trimethylene sulfone and 3-methyl sulfolane,** *Int. J. Chem. Kinet.* **VII,** 799-806 (1975).

15953. Shih, A., Parsegian, V. A., **Van der Waals forces between heavy alkali atoms and gold surfaces: Comparison of measured and predicted values,** *Phys. Rev. A* **12,** No. 3, 835-841 (Sept. 1975).

15954. Epstein, M. S., Rains, T. C., O'Haver, T. C., **Wavelength modulation for background correction in graphite furnace atomic emission spectrometry,** *Appl. Spectrosc.* **30,** No. 3, 324-329 (May-June 1976).

15955. Newman, M., **On a theorem of Cebotarev,** *Linear and Multilinear Algebra* **3,** 259-262 (1976).

15956. Berger, H., **Nondestructive evaluation: A review of nuclear methods,** (Proc. 22nd Int. Instrumentation Symp. on Aerospace Test Measurement, San Diego, CA, May 25-27, 1976), Paper in *22nd International Instrumentation Symposium*, B. Washburn, Ed., **13,** 427-437 (Instrument Society of America, Pittsburgh, PA, 1976).

15957. Domalski, E. S., Tsang, W., **Assessing the hazard potential of chemical substances,** *Proc. 4th Int. Symp. on Transport of Hazardous Cargoes by Sea and Inland Waterways, Jacksonville, FL, Oct. 26-30, 1975,* pp. 259-274 (National Academy of Sciences, Washington, DC, 1975).

15958. Scheer, M. D., **The early afterglow in a pulsed helium discharge,** (Proc. XII Int. Conf. on Phenomena in Ionized Gases, Eindhoven, The Netherlands, Aug. 18-22, 1975), Paper in *Proceedings XII International Conference on Phenomena in Ionized Gases*, J. G. A. Holscher and D. C. Schram, Eds., p. 82 (North-Holland Publishing Co., The Netherlands, 1975).

15959. Lamotte, M., Dewey, H. J., Ritter, J. J., Keller, R. A., **Laser induced photochemical enrichment of chlorine isotopes,** (Proc. 27th Int. Meeting of the Societe De Chimie Physique, on Lasers in Physical Chemistry and Biophysics, Thiais, France, June 17-20, 1975), Paper in *Lasers in Physical Chemistry and Biophysics*, J. Joussot-Dubien, Ed., pp. 153-164 (Elsevier Scientific Publishing Co., Amsterdam, The Netherlands, 1975).

15960. McLaughlin, W. L., **Radiation dosimeter colour,** Article in *Encyclopaedic Dictionary of Physics*, J. Thewlis, Ed., Suppl. 5, 248-251 (Pergamon Press Ltd., Oxford, England, 1975).

15961. Bander, M., Shaw, G. L., Thomas, P., Meshkov, S., **Exotic mesons and e^+e^- annihilation,** *Phys. Rev. Lett.* **36,** No. 13, 695-697 (Mar. 29, 1976).

15962. Karl, G., Meshkov, S., Rosner, J. L., **Symmetries, angular distributions in $\psi' \to \gamma\chi \to \gamma\gamma\psi$, and the interpretation of the $\chi(3400-3550)$ levels,** *Phys. Rev. D* **13,** No. 5, 1203-1215 (Mar. 1, 1976).

15963. Keery, W. J., Leedy, K. O., Galloway, K. F., **Electron beam effects on microelectronic devices,** (Proc. 9th Annual Scanning Electron Microscope Symp., Toronto, Canada, Apr. 5-9, 1976), Paper in *Scanning Electron Microscopy*, O. Johari, Ed., **1,** Part IV, 507-514 (IIT Research Institute, Chicago, IL, 1976).

15964. Shoenfeld, P. S., DeVoe, J. R., **Statistical and mathematical methods in analytical chemistry,** *Anal. Chem.* **48,** No. 5, 403R-411R (Apr. 1976).

15965. Newman, M., **Polynomials in a matrix,** *Linear and Multilinear Algebra* **3,** 263-266 (1976).

Key words: direct sums; formal power series; Jordan canonical form; permutation congruence; polynomials.

Let A be a matrix, $p(x)$ a polynomial. Put $B = p(A)$. It is shown that necessary and sufficient conditions for A to be a polynomial in B are (i) if λ is any eigenvalue of A, and if some elementary divisor of A corresponding to λ is nonlinear, then $p'(\lambda) \neq 0$; and (ii) if λ, μ are distinct eigenvalues of A, then $p(\lambda)$, $p(\mu)$ are also distinct. Here all computations are over some algebraically closed field.

15966. Kuehner, E. C., Cassatt, W. A., **A laser microphotometer for determining uniformity of particulate deposits on membrane filters,** *Appl. Spectrosc.* **30,** No. 3, 289-290 (May-June 1976).

15967. Yakowitz, H., Newbury, D. E., **A simple analytical method for thin film analysis with massive pure element standards,** (Proc. 9th Annual Scanning Electron Microscope Symp., Toronto, Canada, Apr. 5-9, 1976), Paper in *Scanning Electron Microscopy*, O. Johari, Ed., **1,** Part 1, 151-162 (IIT Research Institute, Chicago, IL, 1976).

15968. Dragoo, A. L., Paule, R. C., **Evaporation, diffusion and convection in containerless melts at low gravity,** *AIAA J.* **14,** No. 2, 135-136 (Feb. 1976).

15969. Feldman, A., Horowitz, D., Waxler, R. M., **Stress-optic measurements in the infrared,** (Proc. Third Conf. on High Power Infrared Laser Window Materials, Hyannis, MA, Nov. 12-14, 1973), Paper in *Third Conference on High Power Infrared Laser Window Materials. II. Materials*, C. A. Pitha, A. Armington, and H. Posen, Eds., AFCRL-TR-74-0085(II), Special Reports, No. 174, pp. 403-409 (Air Force Cambridge Research Laboratories, Bedford, MA, Feb. 14, 1974).

15970. Parks, S. I., Johannesen, R. B., **An algorithm for automatic correction of quadrature phase detector outputs,** *J. Magn. Reson.* **22,** 265-267 (1976).

15971. Bennett, H. S., Forman, R. A., **Absorption coefficients of highly transparent solids: Photoacoustic theory for cylindrical configurations,** *Appl. Opt.* **15,** No. 5, 1313-1321 (May 1976).

15972. Hughes, E. E., **Development of standard reference materials for air quality measurement,** *ISA Trans.* **14,** No. 4, 281-291 (1975).

15973. Choi, C. S., Prince, E., **A neutron diffraction study of structure and thermal motion in several monovalent metal azides,** *J. Chem. Phys.* **64,** No. 11, 4510-4516 (June 1, 1976).

15974. Gatehouse, B. M., Negas, T., Roth, R. S., **The crystal structure of *M*-$LiTa_3O_8$ and its relationship to the mineral wodginite,** *J. Solid State Chem.* **18,** 1-7 (1976).

15975. Cameron, J. M., **Traceability?,** *J. Qual. Technol.* **7,** No. 4, 193-195 (Oct. 1975).

15976. Cameron, J. M., **Measurement assurance,** *J. Qual. Technol.* **8,** No. 1, 53-55 (Jan. 1976).

15977. Loevinger, R., Berman, M., **A revised schema for calculating the absorbed dose from biologically distributed radionuclides,** *J. Nucl. Med. MIRD Pamphlet No. 1,* Revised, 3-10 (Mar. 1976).

15978. Taylor, J. L., **Evaluation of data obtained by reference methods,** (Proc. American Society for Testing and Materials Symp. on Calibration in Air Monitoring, Boulder, CO, Aug. 4-8, 1975), *Am. Soc. Test. Mater. Spec. Tech. Publ. 598,* pp. 156-163 (1976).

15979. Taylor, B. N., **Is the present realization of the absolute ampere in error?,** *Metrologia* **12,** No. 2, 81-83 (1976).

15980. Hall, J. L., **Stabilized lasers and the speed of light,** (Proc. 5th Int. Conf. on Atomic Masses and Fundamental Constants, Paris, France, June 1975), Paper in *Atomic Masses and Fundamental Constants* **5,** J. H. Sanders and A. H. Wapstra, Eds., Part 8, 322-329 (Plenum Press, New York, NY, 1976).

15981. Johnson, P. M., **The multiphoton ionization spectrum of trans-1,3-butadiene,** *J. Chem. Phys.* **64,** No. 11, 4638-4644 (June 1, 1976).

15982. Hughes, E. E., **Role of the National Bureau of Standards in calibration problems associated with air pollution measurements,** (Proc. American Society for Testing and Materials Symp. on Calibration in Air Monitoring, Boulder, CO, Aug. 4-8, 1975), *Am. Soc. Test. Mater. Spec. Tech. Publ. 598,* pp. 223-231 (1976).

15983. Heinrich, K. F. J., **Empirical approaches for the treatment of interelement effects,** (Proc. 24th Annual Conf. on Applications of X-Ray Analysis, Denver, CO, Aug. 6-8, 1975), Chapter in *Advances in X-Ray Analysis,* R. W. Gould, C. S. Barrett, J. B. Newkirk, and C. O. Ruud, Eds., **19,** 75-84 (Kendall/Hunt Publ. Co., Dubuque, IA, 1976).

15984. Pella, P. A., Kuehner, E. C., Cassatt, W. A., **Development of a particulate reference sample on membrane filters for the standardization of x-ray fluorescence spectrometers,** (Proc. 24th Annual Conf. on Applications of X-Ray Analysis, Denver, CO, Aug. 6-8, 1975), Chapter in *Advances in X-Ray Analysis,* R. W. Gould, C. S. Barrett, J. B. Newkirk, and C. O. Ruud, Eds., **19,** 463-472 (Kendall/Hunt Publ. Co., Dubuque, IA, 1976).

15985. Siedle, A. R., **Group VIb metal carbonyl complexes of 1,3-dithiolene-2-thione,** *Inorg. Nucl. Chem. Lett.* **11,** No. 5, 345-347 (1975).

15986. Flynn, D. R., **Measurement of thermal conductivity by steady-state methods in which the sample is heated directly by passage of an electric current,** Chapter 5 in *Thermal Conductivity,* R. P. Tye, Ed., **1,** 242-300 (Academic Press, London and NY, 1969).

15987. Linsky, J. L., Glackin, D. L., Chapman, R. D., Neupert, W. M., Thomas, R. J., **The solar XUV spectrum of He II,** *Astrophys. J.* **203,** No. 2, 509-520 (Jan. 15, 1976).

15988. Cook, J. M., Evenson, K. M., Howard, C. J., Curl, R. F., Jr., **Laser magnetic resonance spectrum of HCO on the D_2O 108 μ laser line,** *J. Chem. Phys.* **64,** No. 4, 1381-1388 (Feb. 15, 1976).

15989. Sanders, D. M., Schaefer, H. A., **Reactive vaporization of soda-lime-silica glass melts,** *J. Am. Ceram. Soc.* **59,** No. 3-4, 96-101 (Mar.-Apr. 1976).

15990. Kaufman, V., Radziemski, L. J., Jr., **The sixth spectrum of uranium (U VI),** *J. Opt. Soc. Am.* **66,** No. 6, 599-600 (June 1976).

15991. Miles, M. G., Wager, J. S., Wilson, J. D., Siedle, A. R., **Reactions of 4,5-dicyano-1,3-dithiole-2-thione and -1,3-dithiol-2-one with tervalent phosphorus compounds,** *J. Organ. Chem.* **40,** No. 18, 2577-2582 (1975).

15992. Gutschick, K. A., Clifton, J. R., **Durability study of 14-year old masonry wallettes,** *Am. Soc. Test. Mater. Spec. Tech. Publ. 589, Masonry: Past and Present,* 76-95 (Aug. 1975).

15993. Halsey, N., **GFRP rod and rope for structural guy applications,** *ASCE J. Engr. Mech. Div.* **100,** No. EM4, 673-685 (Aug. 1974).

15994. Zuckerman, B., Turner, B. E., Johnson, D. R., Palmer, P., Morris, M., **A new interstellar line: The 5_1-4_0 (E_2) transition in methyl alcohol,** *Astrophys. J.* **177,** No. 3, 601-607 (Nov. 1, 1972).

15995. Bushnell, D. L., Jr., Huebner, R. H., Celotta, R. J., Mielczarek, S. R., **Preliminary report on electron energy-loss measurements for $CClF_3$, CCl_2F_2, and CCl_3F,** *Argonne Natl. Lab. Report No. ANL-75-60,* Part I. Radiological and Environmental Research Annual Report on Fundamental Molecular Physics and Chemistry, pp. 7-17 (Argonne National Laboratory, Argonne, IL, July 1974-June 1975).

15996. Wallace, B. M., Brown, W. E., **Stoichiometric composition of whitlockite,** *J. Dent. Res.* **50,** No. 2, 343-346 (Mar.-Apr. 1971).

15997. Huebner, R. H., Bushnell, D. L., Jr., Celotta, R. J., Mielczarek, S. R., Kuyatt, C. E., **Ultraviolet photoabsorption by halocarbons 11 and 12 from electron-impact measurements (Summary),** *Argonne Natl. Lab. Report No. ANL-75-60, Part I. Radiological and Environmental Research Annual Report on Fundamental Molecular Physics and Chemistry,* pp. 18-21 (Argonne National Laboratory, Argonne, IL, July 1974-June 1975 .

15998. Huebner, R. H., Celotta, R. J., Mielczarek, S. R., Kuyatt, C. E., **Apparent oscillator strengths for nitrous oxide (Summary),** *Argonne Natl. Lab. Report No. ANL-75-60, Part I. Radiological and Environmental Research Annual Report on Fundamental Molecular Physics and Chemistry,* pp. 3-6 (Argonne National Laboratory, Argonne, IL, July 1974-June 1975).

15999. Huebner, R. H., Celotta, R. J., Mielczarek, S. R., Kuyatt, C. E., **Apparent oscillator strengths for molecular oxygen, derived from electron energy-loss measurements (Summary),** *Argonne Natl. Lab. Report No. ANL-75-60, Part I. Radiological and Environmental Research Division Annual Report on Fundamental Molecular Physics and Chemistry,* pp. 1-2 (Argonne National Laboratory, Argonne, IL, July 1974-June 1975).

16000. Jaeger, E. F., Oster, L., Phelps, A. V., **Growth of thermal constrictions in a weakly ionized gas discharge in helium,** *Phys. Fluids* **19,** No. 6, 819-830 (June 1976).

16001. Youden, W. J., **Evaluation of analytical data,** Chapter in *Encyclopedia of Industrial Chemical Analysis,* F. D. Snell and C. L. Hilton, Eds., **1,** Gen. Tech. A-4, 746-763 (Interscience Publ., New York, NY, 1966).

16002. Steiner, B. W., **Review of the CIE photometric model and of the status of radiometric measurements,** *Proc. Symp. on The Basis for Effective Management of Lighting Energy, Washington, DC, Oct. 29-30, 1976,* pp. 185-195 (Federal Energy Administration, Washington, DC, 1976).

16003. Nimeroff, I., **Symposium on visual signaling,** *Appl. Opt.* **13,** No. 10, 2186, 2445-2446 (Oct. 1974).

16004. Fong, J. T., **Energy approach for creep-fatigue interactions in metals at high temperatures,** *Trans. ASME J. Pressure Vessel Tech.* **97,** Series J, No. 3, 214-222 (1975).

16005. Touloukian, Y. S., Kirby, R. K., Taylor, R. E., Desai, P. D., **Thermal expansion, metallic elements and alloys,** *Thermophysical Properties of Matter, Volume 12,* 1414 pages (Plenum Publishing Corp., New York, NY, 1975).

16006. Stiefel, S. W., **Procedural options for modification of architectural glass in residences to improve occupant safety,** *Proc. 24th Annual Conf. Standards Engineers Society, Syracuse, NY, Sept. 28-Oct. 1, 1975,* pp. 61-64 (Standards Engineers Soc. Inc., Minneapolis, MN, 1975).

16007. Marvin, R. S., Sherwood, G. B., **A guide to sources of information on materials,** Chapter in *Handbook of Materials Science. Volume III: Nonmetallic Materials and Applications,* C. T. Lynch, Ed., pp. 603-627 (CRC Press, Inc., Cleveland, OH, 1975).

16008. Yonemura, G. T., Kohayakawa, Y., **A new look at the research basis for lighting level recommendations,** *Proc. Symp. on The Basis for Effective Management of Lighting Energy, Washington, DC, Oct. 29-30, 1975,* pp. 151-183 (Federal Energy Administration, Washington, DC, 1976).

16009. Walleigh, R. S., **Controlled environments for meaningful measurement,** *Res. Develop. Mag.* **20,** No. 1, 16-21 (Jan. 1969).

16010. Hougen, J. T., **Second all-union symposium on high resolution molecular spectroscopy,** *Appl. Opt.* **14,** No. 1, 8-9 (Jan. 1975).

16011. Weiss, G. H., Rubin, R. J., **The theory of ordered spans of unrestricted random walks,** *J. Stat. Phys.* **14,** No. 4, 333-350 (1976).

16012. Savage, B. D., **High-resolution profiles of the diffuse interstellar feature at 5780 Å,** *Astrophys. J.* **205,** No. 1, 122-135 (Apr. 1, 1976).

16013. Meissner, P., **Evaluation of techniques for verifying personal identity,** *Proc. ACM-NBS Fifteenth Annual Technical Symp., Gaithersburg, MD, June 17, 1976,* pp. 119-127 (Association for Computing Machinery, Washington, DC, 1976).

16014. Bald, J. F., Jr., Sharp, K. G., MacDiarmid, A. G., **The isolation and characterization of pentafluorodisilane,** *J. Fluorine Chem. Short Commun.* **3,** 433-435 (1973/74).

16015. Watson, R. E., Perlman, M. L., Herbst, J. F., **Core level shifts in the 3*d* transition metals and tin,** *Phys. Rev. B* **13,** No. 6, 2358-2365 (Mar. 15, 1976).

16016. Vorburger, T. V., Waclawski, B. J., Sandstrom, D. R., **Improved microwave-discharge source for uv photoemission,** *Rev. Sci. Instrum* **47,** No. 4, 501-504 (Apr. 1976).

16017. McBee, C. L., Kruger, J., **Ellipsometric spectroscopic study of breakdown of Fe-Cr and Fe-Cr-Mo alloys,** *Proc. USA-Japan Seminar on Passivity and Its Breakdown on Iron and Iron Base Alloys, Honolulu, HI, Mar. 10-12, 1975,* pp. 131-132 (National Association of Corrosion Engineers, Houston, TX, 1976).

16018. Kruger, J., **Chemical breakdown of passivity,** *Proc. USA-Japan Seminar on Passivity and Its Breakdown on Iron and Iron Base Alloys, Honolulu, HI, Mar. 10-12, 1975,* pp. 91-98 (National Association of Corrosion Engineers, Houston, TX, 1976).

16019. Moore, R. T., **The influence of ink on the quality of fingerprint impressions,** *Proc. Conf. on the Science of Fingerprints, London, England, Sept. 24-25, 1974,* pp. 69-84 (1974).

16020. Wehrli, R., Bryan, J., **Assemblage: A process-oriented teaching method,** *AIA J.*, pp. 75-77 (May 1976).

16021. Mangum, B. W., Wood, S. D., **Reliability of temperature measurements and control provided in clinical instruments (Abstract),** *Proc. 28th Annual Conf. Engineering in Medicine and Biology, New Orleans, LA, Sept. 20-24, 1975,* p. 251 (1975).

16022. Zalubas, R., Hagan, L., **Atomic energy levels of rare earth elements,** *Proc. 11th Rare Earth Research Conf., Traverse City, MI, Oct. 7-10, 1974,* **1,** 411-416 (1974).

16023. Yates, J. T., Jr., Erickson, N. E., Worley, S. D., Madey, T. E., **The use of x-ray photoelectron spectroscopy (ESCA) for studying adsorbed molecules,** Chapter in *The Physical Basis for Heterogeneous Catalysis,* E. Drauglis and R. I. Jaffee, Eds., pp. 75-105 (Plenum Press, New York, NY, 1975).

16024. Yahalom, J., Ives, L. K., **Electron-microscopical studies of passive films on stainless steels,** *Proc. USA-Japan Seminar on Passivity and Its Breakdown on Iron and Iron Base Alloys, Honolulu, HI, Mar. 10-12, 1975,* pp. 69-71 (National Association of Corrosion Engineers, Houston, TX, 1976).

16025. Cotton, I. W., Benoit, J. W., **Prospects for the standardization of packet-switched networks,** (Proc. Fourth Data Communications Symp., Quebec City, Canada, Oct. 7-9, 1975), Paper in *Network Structures in an Evolving Operational Environment, IEEE Catalog No. 75 CH1001-7 Data,* pp. 2-1–2-7 (The Institute of Electrical and Electronics Engineers Inc., New York, NY, 1975).

16026. Wang, F. W., **Frictional properties of dilute block-copolymer solutions and homopolymer solutions. Application to molecular weight determination,** *Macromolecules* **9,** No. 1, 97-101 (Jan.-Feb. 1976).

16027. Wright, J. R., **Opening remarks by the session chairman,** (Proc. 6th Congress on The Impact of Research on the Built Environment, Budapest, Hungary, Oct. 2-8, 1974), Chapter in *Congress Book II, 60-61 (Conseil International du Batiment Pour la Recherche, L'Etude et la Documentation, Rotterdam, The Netherlands, 1974).*

16028. Van Zyl, B., Chamberlain, G. E., Dunn, G. H., Ruthberg, S., **Definitive pressure generation in the 10^{-4}-10^{-3} torr range for atomic scattering experiments,** *J. Vac. Sci. Technol.* **13,** No. 3, 721-727 (May/June 1976).

16029. Heydemann, P. L. M., **Ultrasonic measurements at pressures up to 50 kbar,** *Proc. Intern. Colloq. Les Propriétés Physiques des Solides Sous Pression, Grenoble, France, Sept. 8-10, 1969,* No. 188, pp. 461-467 (Centre National de la Recherche Scientifique, Paris, France, 1970).

16030. Wegstein, J. H., **The automated classification and identification of fingerprints,** *Proc. Conf. on the Science of Fingerprints, London, England, Sept. 24-25, 1974,* pp. 49-67 (1974).

16031. Haisch, B. M., Linsky, J. L., **Properties of the chromosphere-corona transition region in Capella,** *Astrophys. J.* **205,** No. 1, L39-L42 (Apr. 1, 1976).

16032. Beitler, S. R., Armstrong, G. T., Domalski, E. S., Keane, T. R., Eds., **Book, Combustion Fundamentals for Waste Incineration,** 217 pages (The American Society of Mechanical Engineers, New York, NY, 1974).

16033. Burnett, E. D., **NBS hearing aid test procedures, Chapter III. Performance measurement data from the National Bureau of Standards,** *Handbook of Hearing and Measurement 1976,* **IB 11-52,** pp. 14-299 (Veterans Administration, Washington, DC, 1976).

16034. Evans, A. G., Johnson, H., **The fracture stress and its dependence on slow crack growth,** *J. Mater. Sci.* **10,** 214-222 (1975).

16035. Epstein, G. L., Reader, J., **Resonance lines and energy levels of Cs III, Ba IV, and La V,** *J. Opt. Soc. Am.* **66,** No. 6, 590-598 (June 1976).

16036. Albus, J. S., Evans, J. M., Jr., **A hierarchical structure for robot control,** *Proc. 5th Int. Symp. on Industrial Robots, Chicago, IL, Sept. 22-24, 1975,* pp. 231-238 (Society of Manufacturing Engineers, Dearborn, MI, 1975).

16037. McBee, C. L., Hastie, J. W., **Mechanistic studies of phosphorus containing flame retardants,** *Proc. 1975 Int. Symp. on Flammability and Fire Retardants, Montreal, Canada, May 22-23, 1975,* pp. 253-263 (Technomic Publication Co., Inc., Westport, CT, 1976).

16038. Bagg, T. C., **A technological review: The future of microimagery in the library,** *Drexel Lib. Quart.* **11,** No. 4, 66-74 (Oct. 1975).

16039. Evans, A. G., Fuller, E. R., **Proof testing—The effects of slow crack growth,** *Mater. Sci. Eng.* **19,** No. 1, 69-77 (1975).

16040. Henderson, M. M., **Copyright impacts of future technology,** (Proc. Symp. Centennial Meeting of the American Chemical Society, New York, NY, Apr. 5, 1976), *J. Chem. Inf. Comput. Sci* **16,** No. 2, 72-74 (May 1976).

16041. VanderHart, D. L., **Characterization of the methylene ^{13}C chemical shift tensor in the normal alkane n-$C_{20}H_{42}$,** *J. Chem. Phys.* **64,** No. 2, 830-834 (Jan. 15, 1976).

16042. Norden, B. N., **National measurement system: Length and related dimensional measurements—A micro study,** *NCSL Newsletter* **15,** No. 2, 21-26 (June 1975).

16043. Hamer, W. J., **Standard cells,** Chapter in *The Primary Battery,* G. W. Heise and N. C. Cahoon, Eds., **1,** 433-477 (John Wiley & Sons, Inc., New York, NY, 1971).

16044. Wets, R. J. B., Witzgall, C., **Towards an algebraic characterization of convex polyhedral cones,** *Numer. Math.* **12,** 134-138 (1968).

16045. Pyke, T. N., Jr., **Assuring user service quality in a distributed computer network,** *Proc. COMPCON 76 Spring, San Francisco, CA, Feb. 24-27, 1976,* pp. 73-76 (IEEE Computer Society, Washington, DC, 1976).

16046. Durig, J. R., Lafferty, W. J., Kalasinsky, V. F., **Spectra and structure of small-ring molecules. 33. Microwave spectrum of silacyclopentane,** *J. Phys. Chem.* **80,** No. 11, 1199-1202 (1976).

16047. Albus, J. S., Evans, J. M., Jr., **Robot systems,** *Sci. Am.* **234,** No. 2, 77-86B (Feb. 1976).

16048. Branstad, M. A., Branstad, D. K., Jeffery, S., **Terminals: Out of sight but under control,** *Proc. COMPCON 74, San Francisco, CA, Feb. 26-28, 1974,* pp. 53-55 (IEEE Computer Society, Washington, DC, 1974).

16049. LeVanda, C., Bechgaard, K., Cowan, D. O., Mueller-Westerhoff, U. T., Eilbracht, P., Candela, G. A., Collins, R. L., **Bis(fulvalene)diiron, its mono- and dications. Intramolecular exchange interactions in a rigid system,** *J. Am. Chem. Soc.* **98,** No. 11, 3181-3187 (May 26, 1976).

16050. Ferraris, J. P., Finnegan, T. F., **Electric susceptibility and d.c. conductivity of crystalline TTF–TCNQ,** *Solid State Commun.* **18,** Nos. 9/10, 1169-1172 (1976).

16051. Chang, T. T., Cohen, M. I., **Paramagnetic resonance of Dy^{3+} in CdF_2,** *J. Chem. Phys.* **64,** No. 12, 5255-5260 (June 15, 1976).

16052. Mihalas, D., Shine, R. A., Kunasz, P. B., Hummer, D. G., **Resonance-line transfer with partial redistribution. VIII. Solution in the comoving frame for moving atmospheres,** *Astrophys. J.* **205,** No. 2, 492-498 (Apr. 15, 1976).

16053. Stevens, M. E., **Compatibility problems of network interfacing,** *Proc. Conf. on Interlibrary Communications and Information Networks, Warrenton, VA, Sept. 28-Oct. 2, 1970,* pp. 202-212 (American Library Association, Chicago, IL, 1971).

16054. Duff, J. W., Truhlar, D. G., **Classical *S* matrix: Application of the Bessel uniform approximation to a chemical reaction,** *Chem. Phys. Lett.* **40,** No. 2, 251-256 (June 1, 1976).

16055. Kuriyama, M., Boettinger, W. J., **On the angular divergence of out-going beams in an asymmetric diffraction geometry,** *Acta Cryst. A* **32,** Part 3, 511-512 (May 1976).

16056. Geller, S. B., **Erasing myths about magnetic media,** *Datamation,* pp. 65-68, 70 (Mar. 1976).

16057. Morton, M., Fetters, L. J., **Homogeneous anionic polymerization of unsaturated monomers,** Chapter in *Macromolecular Reviews,* A. Peterlin, M. Goodman, S. Okamura, B. H. Zimm, and H. F. Mark, Eds., **2,** 71-113 (John Wiley & Sons, Inc., New York, NY, 1967).

16058. Saunders, R. D., Ott, W. R., **Spectral irradiance measurements: Effect of uv-produced fluorescence in integrating spheres,** *Appl. Opt.* **15,** No. 4, 827 (Apr. 1976).

16059. Marshall, H. E., **Efficiency impacts of cost sharing on shoreline management,** *Coastal Zone Manage. J.* **2,** No. 4, 369-382 (1976).

16060. McCulloh, K. E., Dibeler, V. H., **Enthalpy of formation of methyl and methylene radicals of photoionization studies of methane and ketene,** *J. Chem. Phys.* **64,** No. 11, 4445-4450 (June 1, 1976).

16061. Thomas, G. A., Schafer, D. E., Wudl, F., Horn, P. M., Rimai, D., Cook, J. W., Glocker, D. A., Skove, M. J., Chu, C. W., Groff, R. P., Gillson, J. L., Wheland, R. C., Melby, L. R., Salamon, M. B., Craven, R. A., De Pasquali, G., Bloch, A. N., Cowan, D. O., Walatka, V. V., Pyle, R. E., Gemmer, R., Poehler, T. O., Johnson, G. R., Miles, M. G., Wilson, J. D., Ferraris, J. P., Finnegan, T. F., Warmack, R. J., Raaen, V. F., Jerome, D., **Electrical conductivity of tetrathiafulvalenium-tetracyanoquinodimethanide (TTF-TCNQ),** *Phys. Rev. B* **13,** No. 11, 5105-5110 (June 1, 1976).

16062. Hughes, C. E., Walker, J. C., **POPSS, a system for modelling and analyzing operating system resource allocation strategies,** *Proc. 3rd Texas Conf. on Computing Systems, Austin, TX, Nov. 7-8, 1974, Paper 74-CHO895-3C,* pp. 3-6-1—3-6-8 (IEEE Computer Society Publ. Office, Long Beach, CA, 1974).

16063. Pyke, T. N., Jr., **Network access techniques: Some recent developments,** *Proc. 3rd Texas Conf. on Computing Systems, Austin, TX, Nov. 7-8, 1974, Paper 74-CHO895-3C,* pp. 2-2-1—2-2-4 (IEEE Computer Society Publ. Office, Long Beach, CA, 1974).

16064. Evans, A. G., Russell, L. R., Richerson, D. W., **Slow crack growth in ceramic materials at elevated temperatures,** *Metall. Trans.* **6A,** 707-716 (Apr. 1975).

16065. Pella, P. A., Kuehner, E. C., Cassatt, W. A., **Development of thin calibration standards for x-ray fluorescence analysis,** *Environmental Protection Technology Series, EPA-600/2-76-126,* 11 pages (Available as PB253252 from the National Technical Information Service, Springfield, VA, 22161, May 1976).

16066. Washer, F. E., **Cameras, optics, and filters for color aerial photography. Lens design for color,** Chapter III in *Manual of Color Aerial Photography, First Edition,* J. T. Smith, Jr., Ed., 94-138 (American Society of Photogrammetry, Falls Church, VA, 1968).

16067. Garvin, D., Domalski, E. S., Hampson, R. F., Wagman, D. D., **Chemistry in the stratosphere,** Chapter 5 in *The Natural Stratosphere of 1974, CIAP Monograph 1,* A. J. Grobecker, Ed., Report No. DOT-TST-75-51, 28 pages (Dept. of Transportation, Washington, DC, Sept. 1975).

16068. Florin, R. E., **Leo Aloysius Wall,** Biographical item for book *American Chemists and Chemical Engineers,* W. D. Miles, Ed., pp. 490-491 (American Chemical Society, Washington, DC, 1976).

16069. Evans, A. G., Wiederhorn, S. M., Linzer, M., Fuller, E. R., Jr., **Proof testing of porcelain insulators and application of acoustic emission,** *Am. Ceram. Soc. Bull.* **54,** No. 6, 576-581 (June 1975).

16070. Weikel, M. K., Miller, T. R., **A summary of a cost analysis of blood banking alternatives,** *Proc. 3rd Annual Conf. of the Society of Government Economists, Washington, DC, Mar. 23, 1973,* W. E. Kost, Ed., pp. 31-37 (1975).

16071. Huffman, J. C., Roth, R. S., Siedle, A. R., **Synthesis of ternary metal sulfide arrays,** *J. Am. Chem. Soc.* **98,** No. 14, 4340-4341 (July 7, 1976).

16072. Culver, C., **Fire loads and live loads in buildings,** *Proc. Int. Conf. on Performance of Building Structures, Glasgow, Scot-*

land, Mar. 31-Apr. 1, 1976, **1**, 111-119 (Pentech Press Ltd., London, England, 1976).

16073. Manning, J. R., **Comparison of matrix inversion, Monte Carlo, and diffusion of probability techniques for calculation of diffusion correlation factors,** (Proc. Symp. on Computer Simulation for Materials Applications, Gaithersburg, MD, Apr. 19-21, 1976), *Nucl. Metall.* **20**, Pt. 1, 109-120 (Apr. 1976).

16074. Farmer, B. L., Eby, R. K., **Calculations of internal friction resulting from molecular defects in hydrocarbon crystals,** (Proc. Symp. on Computer Simulation of Material Applications, Gaithersburg, MD, Apr. 19-21, 1976), *Nucl. Metall.* **20**, Pt. 1, 154-165 (Apr. 1976).

16075. Davis, R. M., **Federal interest in computer utilization by state and local governments,** *The Bureaucrat* **1**, No. 4, 349-356 (Winter 1972).

16076. Henderson, M., Geddes, S., **Automation and the federal library community: Report on a survey,** *Federal Library Committee*, pp. 1-49 (Library of Congress, Washington, DC, June 1973).

16077. McKinney, J. E., Simha, R., **Configurational thermodynamic properties of polymer liquids and glasses. Poly(vinyl acetate). II,** *Macromolecules* **9**, No. 3, 430-441 (May-June 1976).

16078. Stein, R. J., **Angle- and energy-resolved charged particle spectroscopies—A simple way,** *Rev. Sci. Instrum.* **47**, No. 6, 716-719 (June 1976).

16079. Penn, D. R., **Quantitative chemical analysis by ESCA,** *J. Electron Spectrosc. Relat. Phenom.* **9**, 29-40 (1976).

16080. Watson, R. E., Herbst, J. F., Wilkins, J. W., **Core level shifts of rare-gas atoms implanted in the noble metals,** *Phys. Rev. B* **14**, No. 1, 18-25 (July 1, 1976).

16081. Hocken, R., Moldover, M. R., **Ising critical exponents in real fluids: An experiment,** *Phys. Rev. Lett.* **37**, No. 1, 29-32 (July 5, 1976).

16082. Marinenko, G., Huggett, R. J., Friend, D. G., **An instrument with internal calibration for monitoring chlorine residuals in natural waters,** *J. Fish. Res. Board Can.* **33**, No. 4, Part 1, 822-826 (1976).

16083. Snell, J. E., **Comments on drainage system performance,** *Proc. CIB Commission W62 Symp. on Drainage Services in Buildings, National Swedish Institute for Building Research, Stockholm, Sweden, Sept. 24-25, 1973*, pp. 1:1-1:11 (Swedish Council for Building Research, Stockholm, Sweden, 1974).

16084. Siu, M. C. I., **Analysis of edge-heat-loss of a guarded-hot-plate apparatus,** *Proc. 14th Int. Conf. on Thermal Conductivity, Storrs, CT, June 2-4, 1975*, P. G. Klemans and T. K. Chu, Eds., pp. 413-422 (Plenum Press, NY, 1976).

16085. Yates, J. T., Jr., Madey, T. E., Erickson, N. E., Worley, S. D., **Adsorption and condensation of small molecules on tungsten (100) as studied using x-ray photoelectron spectroscopy,** *Chem. Phys. Lett.* **39**, No. 1, 113-117 (Apr. 1, 1976).

16086. Winder, D. R., **Orientation of hexagonal crystals,** *Am. J. Phys.* **38**, No. 3, 319-326 (Mar. 1970).

16087. Culver, C. G., **Comprehensive survey program for fire and live loads in buildings,** *Proc. Public Buildings Service International Conf. on Firesafety in High-Rise Buildings, Seattle, WA, Nov. 18, 1974*, pp. 1-30 (1974).

16088. Oser, H. J., **An average distance,** Problem 75-12 in *Problems and Solutions* **18**, No. 3, 497-501 (July 1976).

16089. Fechter, J. V., Jr., **Product safety and product performance research at the National Bureau of Standards,** *Proc. 6th Congress of the International Ergonomics Association "Old World, New World, One World," University of Maryland, College Park, MD, July 11-16, 1976*, pp. 136-140 (The Human Factors Society, Inc., Santa Monica, CA, July 1976).

16090. Unassigned.

16091. Margulis, S. T., **The ICET testing program for the certification of engineering technicians,** *Prof. Eng.*, pp. 30-31 (Mar. 1976).

16092. Burch, D. M., Peavy, B. A., Powell, F. J., **Experimental validation of the NBS load and indoor temperature prediction model,** *ASHRAE Trans.* **80**, Part 2, 291-310 (1974).

16093. Bordé, C. J., Hall, J. L., Kunasz, C. V., Hummer, D. G., **Saturated absorption line shape: Calculation of the transit-time broadening by a perturbation approach,** *Phys. Rev. A* **14**, No. 1, 236-263 (July 1976).

16094. Cheron, B., Scheps, R., Gallagher, A., **Continuum radiation and potentials of Tl-noble gas molecules,,** *J. Chem. Phys.* **65**, No. 1, 326-335 (July 1, 1976).

16095. Niatel, M. T., Loftus, T. P., Oetzmann, W., **Comparison of exposure standards for ^{60}Co gamma rays,** *Metrologia* **11**, 17-23 (1975).

16096. Guildner, L. A., Johnson, D. P., Jones, F. E., **Vapor pressure of water at its triple point: Highly accurate value,** *Science* **191**, 1261 (Mar. 26, 1976).

16097. Spencer, L. V., Eisenhauer, C., **Reflection of charged-particle beams at near-grazing incidence,** *Trans. Am. Nucl. Soc.* **23**, 609-610 (June 1976).

16098. Novick, R., Sprott, G., Lucatorto, T., **Identification of the lowest metastable autoionizing level in Rb from rf spectroscopic studies,** *Phys. Rev. A* **14**, No. 1, 273-278 (July 1976).

16099. Jeffery, S., Branstad, D. K., **Security considerations in software systems,** *Proc. 3d Texas Conf. on Computing Systems, Austin, TX, Nov. 7-8, 1974, Paper 74-CHO895-3C*, pp. 8-4-1—8-4-4 (IEEE Computer Society Publ. Office, Long Beach, CA, 1974).

16100. Armstrong, G. T., **Reaction calorimetry of fluorine and the thermochemistry of its compounds,** *L'Actualité Chimique*, pp. 5-14 (Feb. 1976).

16101. Beausoliel, R. W., Phillips, C. W., Snell, J. E., **Field investigation of natural gas pipeline accident, Canterbury Woods, Annandale, VA,** *Fire J.* **68**, No. 3, 77-84 (May 1974).

16102. Blevin, W. R., **Black coatings for thermal radiation detectors,** *Proc. Smithsonian-Eppley Symp. on Solar Radiation. Measurements and Instrumentation, Rockville, MD, Nov. 13-15, 1973*, pp. 18-31 (1973).

16103. Buhl, D., Snyder, L. E., Lovas, F. J., Johnson, D. R., **Is there a maser in the silicon monoxide ground state?,** *Astrophys. J.* **201**, No. 1, L29-L31 (Oct. 1, 1975).

16104. Cali, J. P., **Rationale for reference methods in clinical chemistry,** (Proc. Symp. 9th Int. Congress on Clinical Chemistry, Toronto, Canada, July 13-18, 1975), Paper in *Pure and Applied Chemistry* **45**, 63-68 (Pergamon Press, Ltd., Headington Hill Hall, Oxford, OX3 United Kingdom, 1976).

16105. Unassigned.

16106. Horowitz, E., **Standardization of plastics,** *Encycl. Polym. Sci. Technol.* **12,** 771-786 (1970).

16107. Kashiwagi, T., Newman, D. L., **Flame spread over an inclined thin fuel surface,** *Combust. Flame* **26,** 163-177 (1976).

16108. McCaffrey, B. J., Heskestad, G., **A robust bidirectional low-velocity probe for flame and fire application,** *Combust. Flame Brief Communications* **26,** 125-127 (1976).

16109. McNesby, J. R., **New techniques for the measurement of air pollutants,** *Proc. Int. Symp. on Recent Advances in the Assessment of the Health Effects of Environmental Pollution, Paris, France, June 1974,* Paper No. 47, 1-15 (1974).

16110. Mears, T. W., **NBS Standard reference materials: Their characterization and stability consideration,** *Proc. of the SAMA Calibrator Product Class Standard Forum and Stability Testing Workshop, Chicago, IL, Mar. 18-19, 1975,* 10 pages (Scientific Apparatus Makers Association, Washington, DC 20036, 1975).

16111. Robertson, A. F., **Potential heat: Considerations in the application of a new fire test,** *Fire Mater.* **1,** No. 1, 9-13 (Mar. 1976).

16112. Rockett, J. A., **Fire induced gas flow in an enclosure,** *Combust. Sci. Technol.* **12,** 165-175 (1976).

16113. Benjamin, I. A., **Problems in the correlation of small and large scale tests,** *Proc. Int. Symp. on Fire Safety of Combustible Materials, Edinburgh, Scotland, Oct. 15-17, 1975,* **1,** 141-148 (1975).

16114. Eisenhart, C., **Samuel Stanley Wilks,** *Dictionary of Scientific Biography* **XIV,** 381-386 (Charles Scribner's Sons, New York, NY, 1976).

16115. Eisenhart, C., **William John Youden,** *Dictionary of Scientific Biography* **XIV,** 552-557 (Charles Scribner's Sons, New York, NY, 1976).

16116. French, J. C., **The ARPA/IC/NBS Semiconductor Technology Program,** *Proc. Space System Microelectronics Seminar, Los Angeles, CA, Apr. 15-16, 1975,* pp. 189-199 (The Aerospace Corp., Los Angeles, CA, July 31, 1975).

16117. Judd, D. B., Nickerson, D., **Relation between Munsell and Swedish Natural Color System scales,** *J. Opt. Soc. Am.* **65,** No. 1, 85-90 (Jan. 1975).

16118. Kusuda, T., **NBSLD-National Bureau of Standards heating and cooling load determination program,** *APEC J.* **VIII,** No. 6, 4-11 (Winter 1973-1974).

16119. Goldman, A. J., **Discussion,** (Proc. Symp. on Systems Analysis for Social Problems, Gaithersburg, MD, May 26-28, 1969), Paper in *Systems Analysis for Social Problems,* A. Blumstein, et al., Eds., pp. 93-103 (Washington Operations Research Council, Washington, DC, 1970).

16120. Powell, F. J., **Research on energy conservation in building at the National Bureau of Standards,** (Proc. Conf. on Buildings and the Environment, Cary Arboretum of New York Botanical Garden, Millbrook, NY, Nov. 1974), Chapter 6 in *Buildings and the Environment,* R. Goodland, Ed., pp. 114-140 (Cary Arboretum, Millbrook, NY, 1976).

16121. McNesby, J. R., **Air pollution monitoring instrumentation,** *Proc. Naval Environmental Protection Data Base Instrumentation Workshop, Port Hueneme, CA, July 11-12, 1972,* pp. 150-155 (Naval Civil Engineering Laboratory, Port Hueneme, CA, 1972).

16122. Riley, M. E., Truhlar, D. G., **Effects of the Pauli principle on electron scattering by open-shell targets,** *J. Chem. Phys.* **65,** No. 2, 792-800 (July 15, 1976).

16123. Robertson, A. F., **Estimating smoke production during building fires,** *Fire Technol.* **11,** No. 2, 80-94 (May 1975).

16124. Robertson, A. F., **Test method categorization and fire hazard standards,** *ASTM Stand. News* **3,** No. 11, 18-20 (Nov. 1975).

16125. Robertson, A. F., **Two smoke test methods—A comparison of data,** *Fire Technol.* **10,** No. 4, 282-286 (Nov. 1974).

16126. Sinha, S., Bardsley, J. N., **Symmetric charge transfer in low-energy ion-atom collisions,** *Phys. Rev. A* **14,** No. 1, 104-113 (July 1976).

16127. Stein, M. L., **A saddlepoint theorem for self-dual linear systems,** *Linear Algebra and Appl.* **9,** 55-65 (1974).

16128. Walston, W. H., Federman, C., **Fourier transform techniques for the calibration of shock accelerometers,** (Proc. 21st Int. Instrumentation Symp. on Instrumentation in the Aerospace Industry, Philadelphia, PA, May 19-21, 1975), Paper in Vol. 21 *Instrumentation in the Aerospace Industry* and Vol. 12 *Advances in Test Measurement,* B. Washburn, Ed., pp. 315-321 (Instrument Society of America, Pittsburgh, PA, 1975).

16129. Snell, J. E., Achenbach, P. R., Petersen, S. R., **Energy conservation in new housing design,** *Science* **192,** No. 4246, 1305-1311 (June 25, 1976).

16130. McNesby, J. R., **Problems in the measurement of free radical concentrations in the atmosphere,** *Ber. Bunsenges. Phys. Chem.* **78,** No. 2, 158-162 (1974).

16131. Broussalian, V. L., **Risk measurement and safety standards in consumer products,** (Proc. Conf. on Research in Income and Wealth, Washington, DC, Nov. 30-Dec. 1, 1973), Paper in *Household Production and Consumption,* N. E. Terleckyj, Ed., **40,** 491-524 (National Bureau of Economic Research, New York, NY, 1976).

16132. Cooper, J. W., **High-energy dependence of the molecular-hydrogen photoionization cross section,** *Phys. Rev. A* **9,** No. 5, 2236-2237 (May 1974).

16133. Ekberg, J. O., **Term analysis of Fe VI,** *Phys. Scr.* **11,** No. 1, 23-30 (1975).

16134. Epstein, G. L., Reader, J., **Spectrum of doubly ionized yttrium (Y III),** *J. Opt. Soc. Am.* **65,** No. 3, 310-314 (Mar. 1975).

16135. Geist, J., Dewey, H. J., Lind, M. A., **Low-level periodic pulsed energy measurements with an electrically calibrated pyroelectric detector,** *Appl. Phys. Lett.* **28,** No. 4, 171-173 (Feb. 15, 1976).

16136. Meisters, M., **Concepts and trends in polymer fire testing,** *Mod. Plast.* **52,** No. 9, 76-78, 81-82 (Sept. 1975).

16137. Hamilton, W. C., Edmonds, J. W., Tippe, A., Rush, J. J., **Methyl group rotation and the low temperature transition in hexamethylbenzene. A neutron diffraction study,** *Discuss. Faraday Soc.* **48,** 192-204 (1969).

16138. Tsai, D. H., MacDonald, R. A., **Molecular dynamical studies of energy transport in a crystalline solid,** (Proc. Symp. on Computer Simulation for Materials Applications, Gaithersburg, MD, Apr. 10-21, 1976), *Nucl. Metall.* **20,** Pt. 1, 489-509 (Apr. 1976).

16139. Tovey, H., **The development of the National Fire Data System,** *Fire J.* **68,** No. 6, 91-96 (Nov. 1974).

16140. Young, R. D., **Instrumentation for the chemical analysis of manufactured surfaces,** *Ann. CIRP* **24,** No. 2, 549-554 (Aug. 1975).

16141. Edelman, S., **Piezoelectric polymer stress gages,** *Proc. Symp. on Advancements in Instrumentation for Civil Engineering Applications, Kirtland Air Force Base, NM, May 9-10, 1973,* Technical Report No. AFWL-TR-73-186, pp. 221-224 (Air Force Weapons Laboratory, Kirtland Air Force Base, NM, Oct. 1973).

16142. Eliason, L. K., **Standards for door security,** *Archit. Met.* **21,** No. 3, 6-7 (June 1976).

16143. Evans, J. M., Jr., **Position sensors and computer control techniques for automatic control of agricultural vehicles,** *Proc. IFAC Symp. on Automatic Control for Agriculture, Saskatoon, Saskatchewan, Canada, June 18-20, 1974,* Dr. G. C. Zoerb, Ed., Preprint Sec. A-2, pp. 1-5 (Agricultural Engineering, University of Saskatchewan, Saskatoon, Saskatchewan, Canada, 1974).

16144. Krasny, J. F., **Development of flammability standards,** *Proc. 3d Symp. on Textile Flammability, New York, NY, Apr. 16-17, 1975,* pp. 30-46 (LeBlanc Research Corp., East Greenwich, RI, 1975).

16145. McHenry, H. I., **Ship steel weldments for low temperature service,** *Weld. J.* **55,** No. 5, 387-393 (May 1976).

16146. Meissner, P., Evans, J. M., **Computer-aided design in manufacturing,** (Proc. Technology Workshops, Lafayette, IN, Oct. 3-6, 1973 and Columbus, OH, Nov. 7-10, 1973), Paper in *The Automation Research Council,* A. Seireg, Ed., Report No. 5, pp. 289-303 (The American Automatic Control Council, Holmdel, NJ, 07733, Feb. 1974).

16147. Mitchell, R. A., Woolley, R. M., Baker, S. M., **Finite element simulation of weldbonded joint deformation,** (Proc. Symp. on Computer Simulation for Materials Applications, Gaithersburg, MD, Apr. 19-21, 1976), *Nucl. Metall.* **20,** Pt. 2, 1039-1050 (Apr. 1976).

16148. Collard, J. J., **Is the proposed Federal Standard for brakes adequate for police patrol vehicles?,** *Proc. SAE Automotive Engineering Congress and Exposition, Detroit, MI, Feb. 24-28, 1975,* Paper No. 750399, pp. 1-6 (Society of Automotive Engineers, Inc., Warrendale, PA, 1975).

16149. Krasny, J. F., Francis, J. B., **Ranking extinguishability of burning fabrics,** *J. Am. Ass. Text. Chem. Color.* **6,** No. 6, 135-136 (June 1974).

16150. Parker, W. J., **The large-scale fire-test facility at the National Bureau of Standards,** *Proc. Symp. on Full-Scale Fire Tests, Lancaster, PA, Nov. 11-12, 1974,* pp. 45-48 (1976).

16151. Reed, W. P., **Radioisotope and radiopharmaceutical standards,** *Proc. San Diego Biomedical Symp., San Diego, CA, Feb. 6-8, 1974,* **13,** 507-509 (1974).

16152. Powell, F. J., Kusuda, T., Hill, J. E., **Occupant comfort as a decision-making concept for air conditioning of buildings,** (Proc. CIB Commission W45 (Human Requirements) Symp. on Thermal Comfort and Moderate Heat Stress, Watford, England, Sept. 13-15, 1972), Paper in *Thermal Comfort and Moderate Heat Stress,* pp. 191-215 (Her Majesty's Stationery Office, London ECIV 3RN, England, 1973).

16153. Schaefgen, J. R., **Integrating community utilities for resource conservation,** *Proc. Third National Conf. on Complete Water Reuse: Symbiosis as a Means of Abatement for Multi-Media Pollution, Cincinnati, OH, June 27-30, 1976,* pp. 53-59 (1976).

16154. Slater, P. J., **Leaves of trees,** (Proc. 6th Southeastern Conf. on Combinatorics, Graph Theory, and Computing, Boca Raton, FL, Feb. 17-20, 1975), Paper in *Proceedings of the Sixth Southeastern Conference on Combinatorics, Graph Theory, and Computing,* F. Hoffman, R. C. Mullin, R. B. Levow, D. Roselle, R. G. Stanton, and R. S. D. Thomas, Eds., pp. 549-559 (Utilitas Mathematica Publishing Inc., Winnipeg, Manitoba, Canada, 1975).

16155. Thomson, R., Fuller, E., **Crack morphology in relatively brittle crystals,** (Proc. Symp. on Fracture Mechanics of Ceramics, University Park, PA, July 11-13, 1973), Paper in *Fracture Mechanics of Ceramics,* R. C. Bradt, D. P. H. Hasselman, and F. F. Lange, Eds., **1,** 283-295 (1973).

16156. Wachtman, J. B., Jr., **Emerging priorities in ceramic engineering and science—Viewpoint of the scientist-engineer,** (Proc. Conf. on Emerging Priorities in Ceramic Engineering and Science, Alfred, NY, Nov. 4-6, 1974), Paper in *Ceramic Engineering and Science,* V. D. Frechette, L. D. Pye, and J. S. Reed, Eds., pp. 37-50 (Plenum Publishing Corp., New York, NY, 1974).

16157. McNesby, J. R., **The role of standard reference materials in environmental measurements,** *Proc. National Conf. on Health, Environmental Effects, and Control Technology of Energy Use, Washington, DC, Feb. 9-11, 1976, Report No. 600/776-002,* pp. 58-60 (U.S. Environmental Protection Agency, Washington, DC, 1976).

16158. Freeze, P. D., Thomas, D. B., **Studies of noble-metal thermocouple stability at high temperatures,** *NASA CR-135055,* 54 pages (National Aeronautics and Space Administration, Washington, DC, July 1976).

16159. Meier, M. M., **Neutron standards at the National Bureau of Standards,** *Proc. Third Conf. on Application of Small Accelerators, North Texas State University, Denton, TX, Oct. 21-23, 1974,* CONF-741040-P1 **1,** 365-370 (Available from the National Technical Information Service, VA 22161, 1975).

16160. Friauf, R. J., Young, K. F., Merz, W. J., **Properties of ionic crystals,** Paper 9f in *American Institute of Physics Handbook, Third Ed.,* pp. 9-74—9-127 (McGraw-Hill Book Co., New York, NY, 1972).

16161. Frederikse, H. P. R., **Properties of semiconductors,** Paper 9e in *American Institute of Physics Handbook, Third Ed.,* pp. 9-56—9-74 (McGraw-Hill Book Co., New York, NY, 1972).

16162. Frederikse, H. P. R., Slater, J. C., **Electronic properties of solids,** Paper 9c in *American Institute of Physics Handbook, Third Ed.,* pp. 9-26—9-38 (McGraw-Hill Book Co., New York, NY, 1972).

16163. Frederikse, H. P. R., **Structure, melting point, density, and energy gap of simple inorganic compounds,** Paper 9b in *American Institute of Physics Handbook, Third Ed.,* pp. 9-16—9-25 (McGraw-Hill Book Co., New York, NY, 1972).

16164. Wiese, W. L., Glennon, B. M., **Atomic transition probabilities,** Paper 7i in *American Institute of Physics Handbook, Third Ed.,* pp. 7-200—7-209 (McGraw-Hill Book Co., New York, NY, 1972).

16165. Beaty, E. C., Persson, K. B., **Electrical conductions in gases,** Paper 5e in *American Institute of Physics Handbook, Third Ed.,* pp. 5-138—5-139 (McGraw-Hill Book Co., New York, NY, 1972).

16166. Harris, F. K., **Electrical standards,** Paper 5c in *American Institute of Physics Handbook, Third Ed.,* pp. 5-107—5-119 (McGraw-Hill Book Co., New York, NY, 1972).

16167. Hilsenrath, J., **Thermodynamic properties of gases,** Paper 4h in *American Institute of Physics Handbook, Third Ed.,* pp. 4-162—4-204 (McGraw-Hill Book Co., New York, NY, 1972).

16168. Furukawa, G. T., Douglas, T. B., **Heat capacities,** Paper 4e in *American Institute of Physics Handbook, Third Ed.,* pp. 4-105—4-118 (McGraw-Hill Book Co., New York, NY, 1972).

16169. Powell, R. L., Childs, G. E., **Thermal conductivity,** Paper 4g in *American Institute of Physics Handbook, Third Ed.*, pp. 4-142—4-162 (McGraw-Hill Book Co., New York, NY, 1972).

16170. Greenspan, M., **Acoustic properties of liquids,** Paper 3e in *American Institute of Physics Handbook, Third Ed.*, pp. 3-86—3-98 (McGraw-Hill Book Co., New York, NY, 1972).

16171. McKinney, J. E., Lindsay, R., **Density and compressibility of liquids,** Paper 2l in *American Institute of Physics Handbook, Third Ed.*, pp. 2-148—2-187 (McGraw-Hill Book Co., New York, NY, 1972).

16172. Stegun, I. A., **SI units,** Paper 1b in *American Institute of Physics Handbook, Third Ed.*, pp. 1-8—1-13 (McGraw-Hill Book Co., New York, NY, 1972).

16173. Stegun, I. A., Carter, C., **Mathematics bibliography,** Paper 1a in *American Institute of Physics Handbook, Third Ed.*, pp. 1-2—1-8 (McGraw-Hill Book Co., New York, NY, 1972).

16174. Frederikse, H. P. R., Hosler, W. R., Loeffler, N. L., **Testing and test analysis of MHD-materials,** *Proc. NSF-OCR Engineering Workshop on MHD-Materials, Cambridge, MA, Nov. 20-22, 1974*, pp. 138-149 (1974).

16175. Albus, J. S., Evans, J. M., Johnsen, E. G., **A hierarchical concept for man-machine communication,** (Proc. 18th Annual Meeting of the Human Factors Society, Huntsville, AL, Oct. 15-17, 1974), Paper in *Benefits for Mankind*, E. L. Saenger and M. Kirkpatrick, III, Eds., pp. 506-509 (The Human Factors Society, Santa Monica, CA, Oct. 1974).

16176. Lyons, J. W., **Requirements for fire safety of combustible materials in the United States,** *Proc. Int. Symp. on Fire Safety of Combustible Materials, Edinburgh, Scotland, Oct. 15-17, 1975*, **1,** 12-25 (1975).

16177. Blanc, R. P., **Assisting network users with a network access machine,** *Proc. Annual Conf. of the Association for Computing Machinery, San Diego, CA, Nov. 11-13, 1974*, **1,** 74-84 (Association for Computing Machinery, New York, NY, 1974).

16178. Breden, L. H., Meisters, M., **The effect of sample orientation in the smoke density chamber,** *J. Fire Flammability* **7,** 234-247 (Apr. 1976).

16179. Bright, R. G., **Early warning fire detection,** *Proc. Fire Protection in Interior Design, Cleveland, OH, Apr. 10-11, 1975*, pp. 170-184 (1975).

16180. Pease, R. L., Galloway, K. F., Stehlin, R. A., **Radiation damage to integrated injection logic cells,** (Proc. IEEE Nuclear and Space Radiation Effects Conf., Eureka, CA, July 14-17, 1975), *IEEE Trans. Nucl. Sci.* **NS-22,** No. 6, 2600-2604 (Dec. 1975).

16181. Maki, A. G., Freund, S. M., **Laser Stark measurements on OCS including the observation of zero-field-forbidden $\Delta J = 0, \pm 2$ transitions,** *J. Mol. Spectrosc.* **62,** 90-98 (1976).

16182. Caswell, R. S., Coyne, J. J., **Neutron energy deposition spectra studies,** (Proc. 4th Symp. on Microdosimetry, Verbania Pallanza, Italy, Sept. 24-28, 1973), Paper in *Fourth Symposium on Microdosimetry*, J. Booz, H. G. Ebert, R. Eickel, and A. Waker, Eds., EUR 5122 d-e-f, pp. 967-979 (Commission of the European Communities, Luxembourgh, Mar. 1974).

16183. Gross, D., **The measurement and correlation of fire growth in a room,** *Proc. Symp. on Full-Scale Fire Tests, Lancaster, PA, Nov. 11-12, 1974*, pp. 77-87 (Research and Development Center, Lancaster, PA, 1976).

16184. Hust, J. G., Schramm, R. E., **Density and crystallinity measurements of liquid and solid *n*-undecane, *n*-tridecane, and *o*-xylene from 200 to 350K,** *J. Chem. Eng. Data*, **21,** No. 1, 7-11 (1976).

16185. Richmond, J. C., **Courtroom of the future. Court related use of videotape recordings,** (Proc. Seminar on Civil Trial Advocacy, University of Kentucky, Lexington, KY, Jan. 23-24, 1976), Paper in *Report of Seminar on Civil Trial Advocacy*, pp. 43-45 (Office of Continuing Legal Education, University of Kentucky, Lexington, KY, July 1976).

16186. Richmond, J. G., **The courtroom of the future,** (Proc. 8th Annual Crime Countermeasures Conf., Lexington, KY, Apr. 15-19, 1974), Paper in *Proc. 1974 Carnahan and Int. Crime Countermeasures Conf., UKY BU* **105,** 152 (Aug. 1974).

16187. Bright, R. G., **Regulations and standards concerning fire detection systems in the United States,** *Proc. 7th Int. Seminar on Problems in Automatic Fire Detection, Aachen, Germany, Mar. 5-6, 1975*, pp. 47-60 (Institut für Elektrische Nachrichtentechnik, Aachen, Germany, 1975).

16188. Unassigned.

16189. Clifton, J. R., **Protection of reinforcing bars with organic coatings,** *Mater. Perform.* **15,** No. 5, 14-17 (May 1976).

16190. Treado, M. J., **Development of reports and guidelines for law enforcement communications equipment,** (Proc. 8th Annual Crime Countermeasures Conf., Lexington, KY, Apr. 16-19, 1974), Paper in *Proc. 1974 Carnahan and Int. Crime Countermeasures Conf., UKY BU* **105,** 102-112 (Aug. 1974).

16191. Schramm, R. E., Reed, R. P., **Stacking fault energies of fcc Fe-Ni alloys by x-ray diffraction line profile analysis,** *Metall. Trans. A* **7A,** No. 3, 359-363 (Mar. 1976).

16192. McNeil, J. R., Collins, G. J., Persson, K. B., Franzen, D. L., **Ultraviolet laser action from Cu II in the 2500-Å region,** *Appl. Phys. Lett.* **28,** No. 4, 207-209 (Feb. 15, 1976).

16193. Andrews, J. R., Lawton, R. A., **Electrically strobed optical waveform sampling oscilloscope,** *Rev. Sci. Instrum.* **47,** No. 3, 311-313 (Mar. 1976).

16194. Tobler, R. L., Read, D. T., **Fatigue resistance of a uniaxial S-glass/epoxy composite at room and liquid helium temperatures,** *J. Compos. Mater.* **10,** 32-43 (Jan. 1976).

16195. Dalke, J. L., **Metric system versus anthropomorphic units—A bicentennial coup?,** *Am. Metric J.* **4,** No. 3, 121-123 (1976).

16196. Clark, A. F., Haynes, W. M., Deason, V. A., Trapani, R. J., **Low temperature thermal expansion of barium ferrite,** *Cryogenics* **16,** No. 3, 267-270 (May 1976).

16197. Richmond, J. C., **Criteria for evaluating image quality of night vision devices,** (Proc. 8th Annual Crime Countermeasures Conf., Lexington, KY, Apr. 16-19, 1974), Paper in *Proc. 1974 Carnahan and Int. Crime Countermeasures Conf., UKY BU* **105,** 83-94 (Aug. 1974).

16198. Rockett, J. A., **Who will design fire protection of the future,** *Society of Fire Protection Engineers, Technical Report 75-5*, pp. 1-5 (Society of Fire Protection Engineers, Boston, MA, 1975).

16199. Stokesberry, D. P., Hasegawa, S., **Automatic digital microwave hygrometer, model II,** *Rev. Sci. Instrum.* **47,** No. 5, 556-558 (May 1976).

16200. McDonald, D. G., Johnson, E. G., Harris, R. E., **Modeling Josephson junctions,** *Phys. Rev. B* **13,** No. 3, 1028-1031 (Feb. 1, 1976).

16201. Rockar, E. M., Forman, R. A., **Fe^{4+} ($3d^4$) optical absorption in oxidized $SrTiO_3$,** *Phys. Status Solidi A Short Notes* **33,** K91-K95 (1976).

16202. Howard, C. J., Evenson, K. M., **Rate constants for the reactions of OH with ethane and some halogen substituted ethanes at 296 K,** *J. Chem. Phys.* **64,** No. 11, 4303-4306 (June 1, 1976).

16203. Ogburn, F., **Electrodeposition and government laboratories,** *Plating Surf. Finish.* **63,** No. 7, 24-29 (July 1976).

16204. Birky, M. M., **Philosophy of testing for assessment of toxicological aspects of fire exposure,** *J. Combust. Toxicol.* **3,** 5-23 (Feb. 1976).

16205. Rosenstock, H. M., **The measurement of ionization and appearance potentials,** *Int. J. Mass Spectrom. Ion Physics* **20,** 139-190 (1976).

16206. Hust, J. G., **Thermal conductivity standard reference materials, Progress Report,** (Proc. XIII Int. Conf. on Thermal Conductivity, Lake of the Ozarks, MO, Nov. 5-7, 1973), Paper in *Advances in Thermal Conductivity,* R. L. Reisbig and H. J. Sauer, Jr., Eds., pp. 22-24 (1974).

16207. Daney, D. E., **Turbulent natural convection of liquid deuterium, hydrogen and nitrogen within enclosed vessels,** *Int. J. Heat Mass Transfer* **19,** No. 4, 431-441 (Apr. 1976).

16208. Siegwarth, J. D., **A high conductance helium temperature heat switch,** *Cryogenics* **16,** No. 2, 73-76 (Feb. 1976).

16209. Weiss, A. W., **Transition probabilities for ionized atoms,** (Proc. 4th Int. Conf. on Beam-Foil Spectroscopy, Gatlinburg, TN, Sept. 15-19, 1975), Paper in *Beam-Foil Spectroscopy,* I. A. Sellin and D. J. Pegg, Eds., **1,** 51-68 (Plenum Publishing Corp., New York, NY, 1976).

16210. Larsen, E. B., Andrews, J. R., **Using fiber optics in a broadband, sensitive, isotropic antenna—15 kHz to 150 MHz,** *Proc. IEEE Int. Symp. on Electromagnetic Compatibility, Washington, DC, July 13-15, 1976,* pp. 385-389 (Institute of Electrical & Electronics Engineers, Inc., New York, NY, 1976).

16211. Torchia, D. A., Lyerla, J. R., Jr., Quattrone, A. J., **A ^{13}C magnetic resonance study of the helix and coil states of the collagen peptide α1-CB2,** (Proc. Rehovot Symp. on Poly(Amino Acids), Polypeptides, and Proteins and Their Biological Implications, Israel, May 1-8, 1974), Paper in *Peptides, Polypeptides and Proteins,* E. R. Blout, F. A. Bovey, M. Goodman, and N. Lotan, Eds., pp. 436-448 (John Wiley & Sons, New York, NY, 1974).

16212. Sweeney, W. T., **Dental research at the National Bureau of Standards 1919-1969,** *Br. Dent. J.* **127,** 289-290 (Sept. 16, 1969).

16213. Coriell, S. R., Sekerka, R. F., **The effect of the anisotropy of surface tension and interface kinetics on morphological stability,** *J. Cryst. Growth* **34,** No. 2, 157-163 (July 1976).

16214. Truhlar, D. G., **Interpretation of ortho-para hydrogen conversion,** *J. Chem. Phys.* **65,** No. 3, 1008-1010 (Aug. 1, 1976).

16215. Persensky, J. J., **Human factors in product safety research,** *ASHRAE Trans.* **81,** Part 2, 434-440 (Dec. 1975).

16216. McBee, C. L., Kruger, J., **Events leading to the initiation of the pitting of iron,** (Proc. Int. Conf. on Localized Corrosion, Williamsburg, VA, Dec. 6-10, 1971), Paper in *Localized Corrosion,* R. Staehle, B. F. Brown, J. Kruger, and A. Agrawal, Eds., NACE-3. 252-260 (National Association of Corrosion Engineers, Houston, TX, 1974).

16217. Scheps, R., Gallagher, A., **Teratomic recombination in alkali metal-noble gas vapors,** *J. Chem. Phys.* **65,** No. 3, 859-871 (Aug. 1, 1976).

16218. Fanconi, B., **On the assignments of rotatory lattice modes in *n*-alkanes and polyethylene,** *J. Appl. Phys.* **46,** No. 10, 4124-4129 (Oct. 1975).

16219. Reneker, D. H., Fanconi, B., **Effects of defects on the longitudinal acoustic mode of *n*-alkane chains,** *J. Appl. Phys.* **46,** No. 10, 4144-4147 (Oct. 1975 .

16220. Kusuda, T., Tsuchiya, T., Powell, F. J., **Prediction of indoor temperature by using equivalent thermal mass response factors,** (Proc. 5th Symp. on Temperature, Washington, DC, June 21-24, 1971), Paper in *Temperature, Its Measurement and Control in Science and Industry,* L. G. Rubin, A. C. Anderson, J. E. Janssen, and R. D. Cutkosky, Eds., Part 2, **4,** 1345-1355 (Instrument Society of America, Pittsburgh, PA, 1972).

16221. Moore, L. J., Gramlich, J. W., Machlan, L. A., **Application of isotope dilution to the high accuracy trace analysis of environmental and health standards,** (Proc. 9th Annual Conf. on Trace Substances in Environmental Health, Columbia, MO, June 11, 1975), Paper in *Trace Substances in Environmental Health-IX,* D. D. Hemphill, Ed., pp. 311-316 (1975).

16222. Luther, G. G., Towler, W. R., Deslattes, R. D., Lowry, R., Beams, J., **Initial results from a new measurement of the Newtonian gravitational constant,** (Proc. Atomic Masses and Fundamental Constants, Paris, France, June 2-6, 1975), Paper in *Atomic Masses and Fundamental Constants,* J. H. Sanders and A. H. Wapstra, Eds., **5,** 592-598 (Plenum Publishing Corp., New York, NY, 1976).

16223. Deslattes, R. D., **Recent estimates of the Avogadro constant,** (Proc. Atomic Masses and Fundamental Constants, Paris, France, June 2-6, 1975), Paper in *Atomic Masses and Fundamental Constants,* J. H. Sanders and A. H. Wapstra, Eds., **5,** 552-558 (Plenum Publishing Corp., New York, NY, 1976).

16224. McCarter, R. J., **Smoldering of flexible polyurethane foam,** *J. Consum. Prod. Flammability* **3,** 128-140 (June 1976).

16225. Horowitz, E., **Plastics testing,** Paper in *Kirk-Othmer Encyclopedia of Chemical Technology, 2d Edition,* **15,** 811-832 (John Wiley & Sons, Inc., New York, NY, 1968).

16226. Snell, J. E., **Summary Review—Evaluation factors, design requirements and needed research,** (Proc. Symp. CIB (Conseil Internationale du Batiment) Commission W62 on Water Supply and Drainage Inside Buildings, Watford, England, Sept. 19-20, 1972), Paper in *Water Demand in Buildings,* pp. 117-126 (Dept. of the Environment, Watford, England, Nov. 1973).

16227. Deslattes, R. D., Kessler, E. G., Sauder, W. C., Henins, A., **Visible to gamma-ray wavelength ratio,** (Proc. Atomic Masses and Fundamental Constants, Paris, France, June 2-6, 1975), Paper in *Atomic Masses and Fundamental Constants,* J. H. Sanders and A. H. Wapstra, Eds., **5,** 48-53 (Plenum Publishing Corp., New York, NY, 1976).

16228. Melmed, A. J., Carroll, J. J., **Three-color direct superposition for field-ion microscopy,** *J. Appl. Phys. Commun.* **47,** No. 8, 3762-3763 (Aug. 1976).

16229. Crawford, M. L., **Improved techniques and instrumentation for EMC measurements,** *Proc. IEEE Int. Symp. on Electromagnetic Compatibility, Washington, DC, July 13-15, 1976,* pp. 369-374 (Institute of Electrical and Electronics Engineers, Inc., New York, NY, 1976).

16230. Brown, W. E., Gregory, T. M., **Calcium pyrophosphate crystal chemistry,** *Arthritis Rheum.* **19,** No. 3, 446-462 (May-June 1976).

16231. Barbrow, L. E., **Changeover to metric—Timetable and Government role,** Paper in *Metrication—Managing the Industrial Transition,* R. G. Liptai and J. W. Pearson, Eds., Am. Soc. Test. Mater. Spec. Tech. Publ. 574, pp. 22-31 (July 1975).

16232. Arp, V., **Negative differential flow resistance in supercritical helium,** *Cryogenics* **16,** No. 3, 171-177 (Mar. 1976).

16233. Benjamin, I. A., Adams, C. H., **The flooring radiant panel test and proposed criteria,** *Fire J.* **70,** No. 2, 63-70 (Mar. 1976).

16234. Haller, F. B., Hessel, M. M., Neef, W., Lai, W., Lohr, H., **Concentric heat pipe cavity for e-beam excited lasers,** *Proc. 6th Symp. on Engineering Problems of Fusion Research, San Diego, CA, Nov. 17-21, 1975,* pp. 79-83 (Institute of Electrical and Electronics Engineers, Inc., New York, NY, 1976).

16235. Torchia, D. A., Lyerla, J. R., Jr., Deber, C. M., **Carbon-13 chemical shifts of amides and imino acid residues. Effects of the carbonyl substituent and** *syn-anti* geometries, *J. Am. Chem. Soc. Commun. to Editor* **96,** No. 15, 5009-5011 (July 24, 1974).

16236. Allan, D. W., Mungall, A. G., Winkler, G. M. R., **Comments on the 25 April 1975 paper in "Science" entitled: "Terrestrial timekeeping and general relativity—A discovery,"** *Science* **191,** 489-490 (Feb. 6, 1976).

16237. Jones, R. N., **Electronic eavesdropping techniques and equipment,** *LESP-RPT-0207.00,* 40 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Sept. 1975).

16238. Richmond, J. C., **Active night vision devices,** *NILECJ-STD-0305.00,* 31 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16239. Richmond, J. C., **Passive, first generation night vision devices,** *NILECJ-STD-0304.00,* 25 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16240. Bennett, L. H., Page, C. H., Swartzendruber, L. J., **Comments on units in magnetism,** (Proc. 21st Annual AIP Conf. No. 29, on Magnetism and Magnetic Materials-1975, Philadelphia, PA, Dec. 9-12, 1975), Paper in *Magnetism and Magnetic Materials-1975,* J. J. Becker, G. H. Lander, and J. J. Rhyne, Eds., 3 pages (American Institute of Physics, New York, NY, 1975).

16241. Ramsburg, R. E., Aronoff, M. J., **Calculation by digital computer of minimum station-to-station distances in the United States railroad network,** *Report No. DOT P 6300.1,* 46 pages (Department of Transportation, Washington, DC, June 1972).

16241A. Ramsburg, R. E., Batten, D. R., **Programmer level documentation of the short line mileage programs for the United States railroad network,** *Report No. DOT P 6300.1,* 106 pages (Department of Transportation, Washington, DC, June 1972).

16242. Hougen, J. T., **Calculation of vibrational level populations in multiphoton absorption processes,** *J. Chem. Phys.* **65,** No. 3, 1035-1041 (Aug. 1, 1976).

16243. Wachtman, J. B., Jr., **Ceramic materials in high temperature gases: Fundamental behavior controlling lifetime in coal conversion applications,** (Proc. Ceramic Materials in High Temperature Gases on Materials Problems and Research Opportunities in Coal Conversion, Columbus, OH, Apr. 16-18, 1974), Paper in *Materials Problems and Research Opportunities in Coal Conversion* **2,** 335-343 (The Department of Metallurgical Engineering, The Ohio State University, Columbus, OH, 1974).

16244. Bovey, F. A., Schilling, F. C., McCrackin, F. L., Wagner, H. L., **Short-chain and long-chain branching in low-density polyethylene,** *Macromolecules* **9,** No. 1, 76-80 (Jan.-Feb. 1976).

16245. Davis, R. M., **National policies for information processing: Situation in the United States,** (Proc. 2d Jerusalem Conf. on Information Technology, Jerusalem, Israel, July 29-Aug. 1, 1974), Paper in *The 2d Jerusalem Conference on Information Technology: Computers for Social and Economic Development,* C. C. Gotlieb and H. Maisel, Eds., **1,** 15-27 (1974).

16246. Linden, T. A., **The use of abstract data types to simplify program modifications,** (Proc. Conf. on Data: Abstraction Definition and Structure, Salt Lake City, Utah, Mar. 22-24, 1976), *SIGPLAN Notices 11, Special Issue, Assoc. Comput. Mach.-FDT* **8,** No. 2, 12-23 (Association for Computing Machinery, New York, NY, Mar. 1976).

16247. Abrams, M. D., **Measuring service delivered in interactive computing,** (Proc. 2d Jerusalem Conf. on Information Technology, Jerusalem, Israel, July 29-Aug. 1, 1974), Paper in *The 2d Jerusalem Conference on Information Technology: Computers for Social and Economic Development,* C. C. Gotlieb and H. Maisel, Eds., **1,** 747-754 (1974).

16248. Muehlhause, C. O., **Costs, real and perceived, examined in a risk-benefit decision framework,** (Proc. OECD Conf. on Costs, Real and Perceived, Examined in a Risk-Benefit Decision Framework, Paris, France, Aug. 1972), Paper in *Environmental Damage Costs,* pp. 282-304 (Director of Information, OECD, Paris, France, 1974).

16249. Casella, R. C., **Possible failure of the Pomeranchuk Theorem. Shrinkage of the forward elastic peak and oscillations,** (Proc. "Ettore Majorana" School of Subnuclear Physics, Erice, Sicily, July 1-19, 1970), Paper in *Elementary Processes at High Energy,* A. Zichichi, Ed., 519-525 (Academic Press, Inc., New York, NY, 1971).

16250. Muehlhause, C. O., **Risk benefit analysis,** *ASTM Stand. News* **1,** No. 2, 8-13 (Feb. 1973).

16251. Finnegan, T. F., **Superconductivity in dc voltage metrology,** (Proc. Summer Course on the Science and Technology of Superconductivity, Georgetown University, Washington, DC, Aug. 13-26, 1971), Paper in *The Science and Technology of Superconductivity,* W. D. Gregory, W. N. Mathews, Jr., and E. A. Edelsack, Eds., **2,** 565-585 (Plenum Press, New York, NY, 1973).

16252. Eitzen, D. G., Chwirut, D. J., **Ultrasonic reference blocks and characterized fatigue cracks,** (Proc. ARPA/AFML Review of Quantitative Nondestructive Evaluation, Thousand Oaks, CA, July 15-17, 1975), Paper in *Proceedings of the ARPA/AFML Review of Quantitative NDE, Technical Report AFML-TR-75-212,* pp. 107-127 (Air Force Materials Laboratory, Wright-Patterson Air Force Base, OH, Jan. 1976).

16253. Langhoff, P. W., Corcoran, C. T., **Tchebycheff-derivative approximations to photoabsorption cross sections in atoms and ions,** *Chem. Phys. Lett.* **40,** No. 3, 367-371 (June 15, 1976).

16254. Unassigned.

16255. Deslattes, R. D., Henins, A., Schoonover, R. M., Carroll, C. L., Bowman, H. A., **Avogadro constant—Corrections to an earli-**

er report, *Phys. Rev. Lett. Comments* **36,** No. 15, 898-900 (Apr. 12, 1976).

16256. Bulos, B. R., Phelps, A. V., **Excitation of the 4.3-μm bands of CO_2 by low-energy electrons,** *Phys. Rev. A* **14,** No. 2, 615-629 (Aug. 1976).

16257. Ayres, T. R., Linsky, J. L., **The Mg II *h* and *k* lines. II. Comparison with synthesized profiles and Ca II K,** *Astrophys. J.* **205,** No. 3, Part 1, 874-894 (May 1, 1976).

16258. Schnaus, U. E., Schroeder, J., Haus, J. W., **Subcritical viscosity anomaly in sodium silicate glasses,** *Phys. Lett.* **57A,** No. 1, 92-94 (May 17, 1976).

16259. Mann, W. B., Huchinson, J. M. R., **The standardization of iodine-129 by beta-photon coincidence counting,** *Int. J. Appl. Radiat. Isot. Tech. Notes* **27,** No. 3, 187-188 (Mar. 1976).

16260. Fromhold, A. T., Jr., **Parabolic oxidation of metals in homogeneous electric fields,** *J. Phys. Chem. Solids* **33,** 95-120 (1972).

16261. Carlsten, J. L., Szöke, A., **Collisional redistribution of near-resonant scattered light in Sr vapour,** *J. Phys. B Letter to Editor* **9,** No. 9, L231-L235 (1976).

16262. Chen, S. T., Gallagher, A., **Excitation of the Ba and Ba^+ resonance lines by electron impact on Ba atoms,** *Phys. Rev. A* **14,** No. 2, 593-601 (Aug. 1976).

16263. Suzuki, G., Carter, C., Lieblein, J., Murphey, W. M., Perella, F. J., Smith, C. N., **Safeguards accountability analysis in a nuclear fuel processing plant,** Paper in *Safeguards Systems Analysis of Nuclear Fuel Cycles, WASH-1140 Safeguards and Nuclear Materials Management UC-15,* 46 pages (Office of Safeguards and Materials Management, U.S. Atomic Energy Commission, Washington, DC, Oct. 1, 1969).

16264. Smith, J. C., **The elastic constants of a particulate-filled glassy polymer: Comparison of experimental values with theoretical predictions,** *Polym. Eng. Sci.* **16,** No. 6, 394-399 (June 1976).

16265. Calvano, N. J., Dobbyn, R. C., **Riot helmets,** *NILECJ-STD-0104.00,* 15 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Oct. 1974).

16266. Fey, R. L., **Automatic vehicle location techniques for law enforcement use,** *LESP-RPT-0205.00,* 27 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Sept. 1974).

16267. Robertson, B., Baumgarten, G. P., **Evaluation of automotive fuel flowmeters,** (Proc. Project Coordination Meeting of the Automotive Energy Efficiency Program, Cambridge, MA, Nov. 4-6, 1975), *Report No. DOT-TSC-OST-76-1,* 20 pages (Available from the National Technical Information Service, Springfield, VA 22161, June 1976).

16268. Eicke, W. G., Jr., **The volt, yesterday, today and tomorrow,** (Proc. PSA Int. Conf., Chicago, IL, Oct. 4-7, 1971), *ISA Reprint 71-703,* pp. 1-7 (Oct. 1971).

16269. Cook, R. K., **Acoustic emissions—Researchers in Japan,** *Sci. Bull.* **1,** No. 1, 17 pages (June-Sept. 1976).

16270. Fuller, E. G., Hayward, E., Eds., **Photonuclear reactions,** *Book—Benchmark Papers in Nuclear Physics* **2,** 426 pages (Dowden, Hutchinson & Ross, Inc., Stroudsburg, PA, 1976).

16271. Panek, R. J., Savage, B. D., **Ultraviolet photometry from the orbiting astronomical observatory. XXIII. The resonance lines of triply ionized carbon and silicon in the spectra of hot stars,** *Astrophys. J.* **206,** No. 1, 167-181 (May 15, 1976).

16272. Mihalas, D., Kunasz, P. B., Hummer, D. G., **Solution of the comoving-frame equation of transfer in spherically symmetric flows. III. Effect of aberration and advection terms,** *Astrophys. J.* **206,** No. 2, 515-524 (June 1, 1976).

16273. Swartzendruber, L. J., Evans, B. J., **Nuclear gamma ray resonance observation of the activation process in Raney iron and Urushibara iron catalysts,** *J. Catal.* **43,** 207-219 (1976).

16274. Peterlin, A., **Crystallization from a strained melt or solution,** *Polym. Eng. Sci.* **16,** No. 3, 126-137 (Mar. 1976).

16275. Phillips, J. C., Mattamal, M. M., **Correlation of liquid heat capacities for carboxylic esters,** *J. Chem. Eng. Data* **21,** No. 2, 228-232 (Apr. 1976).

16276. Unassigned.

16277. Mopsik, F. I., Broadhurst, M. G., **Molecular dipole electrets,** *J. Appl. Phys.* **46,** No. 10, 4204-4208 (Oct. 1975).

16278. Penn, R. W., Kearsley, E. A., **The scaling law for finite torsion of elastic cylinders,** *Trans. Soc. Rheol.* **20,** No. 2, 227-238 (1976).

16279. Fanconi, B., Crissman, J., **Influence of methyl group branches on the longitudinal acoustical mode frequencies in alkanes and polyethylene,** *J. Polym. Sci. Polym. Lett. Ed.* **13,** 421-426 (1975).

16280. Horowitz, E., **Transfer of measurement methodology to industry,** *J. Assoc. Eng. Archit. Isr.* **35,** 21-41 (May 1976).

16281. MacDonald, R. A., Tsai, D. H., **Molecular dynamical calculations of the thermal diffusivity of a perfect lattice,** (Proc. 14th Int. Conf. on Thermal Conductivity, Storrs, CT, June 2-4, 1975), Paper in *Proceedings of the 14th International Conference on Thermal Conductivity,* P. G. Klemens and T. K. Chu, Eds., pp. 145-152 (Plenum Press, New York, NY, 1976).

16282. Bucaro, J. A., Simmons, J. H., **Rayleigh wave study of surface diffusion and stress relaxation in glass,** *J. Non-Cryst. Solids* **14,** 225-241 (1974).

16283. DiMarzio, E. A., **Validity of the Ehrenfest relation for a system with more than one order parameter,** *J. Appl. Phys.* **45,** No. 10, 4143-4145 (Oct. 1974).

16284. Broadhurst, M. G., Harris, W. P., Mopsik, F. I., Malmberg, C. G., **Piezoelectricity, pyroelectricity and electrostriction in polymers,** *Polym. Prepr.* **14,** No. 2, 820-829 (Aug. 1973).

16285. Taggart, H., Nelson, R. E., Scott, W., Jr., Shafer, J., Tary, J., Workman, J., Treado, M. J., **Personal/portable FM transmitters,** *NILECJ-STD-0203.00,* 22 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Oct. 1974).

16286. Treado, M. J., Taggart, H. E., Nelson, E., Scott, W. W., Shafer, J. F., Tary, J. J., **Fixed and base station FM transmitters,** *NILECJ-STD-0201.00,* 20 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Sept. 1974).

16287. Calvano, N. J., Dobbyn, R. C., **Crash helmets,** *NILECJ-STD-0105.00,* 13 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16288. Hartman, A. W., **Proposed image space scanner with reduced off-axis defocusing,** *Opt. Eng.* **15,** No. 3, 262-264 (May-June 1976).

16289. Crissman, J. M., **On the transition from paraffinic to polymeric behavior: Mechanical properties,** *J. Polym. Sci. Part A-2,* **13,** 1407-1416 (1975).

16290. DiMarzio, E. A., Bishop, M., **Connection between the macroscopic electric and mechanical susceptibilities,** *J. Chem. Phys.* **60,** No. 10, 3802-3811 (May 15, 1974).

16291. Edelman, S., **Piezoelectric polymer transducers,** *Proc. Workshop on Sonar Transducer Materials, Naval Research Laboratory, Washington, DC, Nov. 13-14, 1975,* pp. 269-282 (Naval Research Laboratory, Washington, DC, Feb. 1976).

16292. Tighe, N. J., Swann, P. R., **Magnetite lamellae in reduced hematites,** Chapter 4.3 in *Electron Microscopy in Mineralogy,* H. R. Wenk, et al., Eds., pp. 209-213 (Springer-Verlag Berlin Heidelberg, Germany, 1976).

16293. Arthur, M. G., Saulsbery, L. F., Scott, W. W., Jr., Shafer, J. F., **Mobile FM receivers,** *NILECJ-STD-0207.00,* 18 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16294. Jickling, R. M., **RF coaxial cable assemblies for mobile transceivers,** *NILECJ-STD-0212.00,* 9 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC. Sept. 1975).

16295. Calvano, N. J., Dobbyn, R. C., **Ballistic helmets,** *NILECJ-STD-0106.00,* 13 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Sept. 1975).

16296. Jesch, R. L., Scott, W. W., Treado, M. J., **Batteries for personal/portable transceivers,** *NILECJ-STD-0211.00,* 14 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16297. Arthur, M. G., Saulsbery, L. F., Scott, W. W., Jr., Shafer, J. F., **Personal/portable FM receivers,** *NILECJ-STD-0208.00,* 18 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Oct. 1975).

16298. Teague, M. R., Lambropoulos, P., Goodmanson, D., Norcross, D. W., **Theory of two-photon ionization of cesium,** *Phys. Rev. A* **14,** No. 3, 1057-1064 (Sept. 1976).

16299. Cooper, J. W., **Inner shell ionization as a single electron process,** (Proc. Int. Conf. on Inner Shell Ionization Phenomena and Future Applications, Atlanta, GA, Apr. 17-22, 1972), Paper in *Proceedings of the International Conference on Inner Shell Ionization Phenomena and Future Applications,* R. W. Fink S. T. Manson, J. M. Palms, and P. V. Rao, Eds., CONF-720404, **2,** 807-815 (USAEC Technical Information Center, Oak Ridge, TN, Jan. 1973).

16300. Davis, R. M., **Preventive technology: A cure for Scientific ills,** *Science* **188,** No. 4185, 213 (Apr. 1975).

16301. Berger, H., **Nondestructive measurements: How good are they?,** *Mater. Eval.* **XXXIV,** No. 1, 18A-20A, 22A, 24A, 33A-34A (Jan. 1976).

16302. Franklin, A. D., **Electrochemical growth of crystals from electrolyte solutions,** *J. Cryst. Growth* **34,** 245-247 (1976).

16303. Arthur, M. G., Saulsbery, L. F., Scott, W. W., Jr., Shafer, J. F., **Fixed and base station FM receivers,** *NILECJ-STD-0206.00,* 17 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Sept. 1975).

16304. Fanconi, B., Finegold, L., **Vibrational states of the biopolymer polyglycine II: Theory and experiment,** *Science* **190,** 458-460 (Oct. 31, 1975).

16305. Dobbyn, R. C., Bruchey, W. J., Jr., Shubin, L. D., **An evaluation of police handgun ammunition: Summary report,** *LESP-RPT-0101.01,* 21 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Oct. 1975).

16306. Lawn, B. R., Swain, M. V., Phillips, K., **On the mode of chipping fracture in brittle solids,** *J. Mater. Sci. Lett.* **10,** 1236-1239 (1975).

16307. Brown, W. E., Chow, L. C., **Chemical properties of bone mineral,** *Ann. Rev. Mater. Sci.* **6,** 213-236 (Aug. 1976).

16308. Horowitz, E., **Metallo-organic systems with high thermal stability,** Paper 7 in *Advances in Chemistry Series,* R. F. Gould, Ed., No. 85, pp. 82-94 (1968).

16309. Chang, S. S., **Low temperature thermal behavior of glassy polymers,** (Proc. ACS Coatings and Plastics 170th Meeting, Chicago, IL, Aug. 25-29, 1975), Paper in *Coatings and Plastics Preprints,* R. F. Wint, Ed., **35,** No. 2, 364-369 (American Chemical Society, Washington, DC, 1975).

16310. Berger, R. E., **Dynamic strain gage measurements on ophthalmic lenses impacted by low energy missiles,** *J. Opt. Physiolog. Opt.* **53,** No. 6, 279-286 (June 1976).

16311. Berger, R. E., **The role of edge failures in impact testing of ophthalmic lenses,** *J. Am. Optometric Assoc.* **47,** No. 5, 599-605 (1976).

16312. Bennett, L. H., Kahan, D. J., Carter, G. C., **Alloy phase diagram data,** *Mater. Sci. Eng.* **24,** 1-17 (1976).

16313. Wardell, J. R., Wilmot, G. B., Haar, L., Klein, M., **Thermodynamic data for interior ballistics calculations,** *Proc. 12th JANNAF Combustion Meeting, Newport, RI, July 25, 1975,* pp. 363-390 (Chemical Propulsion Information Agency, Johns Hopkins University, Applied Physics Laboratory, Laurel, MD, 1976).

16314. Bur, A. J., **Measurements on the dynamic piezoelectric properties of bone as a function of temperature and humidity,** *J. Biomech.* **9,** 495-507 (Sept. 1976).

16315. Pierman, B. C., Bryson, J. O., **Building safety research at the National Bureau of Standards,** *Prof. Safety Mag.,* pp. 13-16 (Sept. 1976).

16316. Margolis, S. A., Howell, B. F., Schaffer, R., **Purification and analysis of the purity of NADH,** *Clin. Chem.* **22,** No. 8, 1322-1329 (1976).

16317. Clark, E. J., Ballard, D. B., Embree, E. J., **SEM examination of an overheated aluminum wired electrical receptacle,** (Proc. 34th Annual Electron Microscopy Society of America Meeting, Miami Beach, FL, Aug. 9-13, 1976), Paper in *34th Annual Proceedings Electron Microscopy Society of America,* G. W. Bailey, Ed., pp. 454-455 (Claitor's Publ. Div., Baton Rouge, LA, 1976).

16318. Waterhouse, R. V., Cook, R. K., **Diffuse sound fields: Eigenmode and free-wave models,** *J. Acoust. Soc. Am.* **59,** No. 3, 576-581 (Mar. 1976).

16319. Davis, R. M., **The computer serves, the consumer relishes,** *Comput. Dec.*, pp. 31-34 (May 1975).

16320. Wiese, W. L., Younger, S. M., **Atomic oscillator strengths in fusion plasma research,** (Proc. IVth Int. Conf. on Beam-Foil Spectroscopy, Gatlinburg, TN, Sept. 15-19, 1975), Paper in *Beam-Foil Spectroscopy*, I. A. Sellin and D. J. Pegg, Eds., **2,** 951-960 (Plenum Publ. Corp., New York, NY, 1976).

16321. Schneider, S. J., Jr., **Materials properties determination: Transport properties, phase equilibria and slag properties,** *Proc. NSF-OCR Engineering Workshop on MHD Materials, Cambridge, MA, Nov. 20-22, 1974,* pp. 150-167 (Massachusetts Institute of Technology, Cambridge, MA, 1975).

16322. Klose, J. Z., **Measurement of atomic uranium lifetimes for isotope separation,** (Proc. SPIE/SPSE Tech. Symp. on Ultra High Power Lasers for Practicable Applications, Reston, VA, Mar. 22-23, 1976), *SPIE 76,* 150-155 (Society of Photo-Optical Instrumentation Engineers, Palos Verdes Estates, CA, 1976).

16323. Unassigned.

16324. Christ, B. W., Swanson, S. R., **Alignment problems in the tensile test,** *J. Test. Eval.* **4,** No. 6, 405-417 (Nov. 1976).

16325. DeSpain, H. E., Garfinkel, S. B., **Calibration of detectors for argon-41,** *Proc. Noble Gases Symp., Las Vegas, NV, Sept. 24-28, 1973,* pp. 155-159 (1973).

16326. Mamrak, S. A., **A predictive response time monitor for computer networks,** *Proc. 3d Int. Conf. on Computer Communication, Toronto, Canada, Aug. 3-6, 1976,* pp. 626-630 (1976).

16327. Lyerla, J. R., Jr., VanderHart, D. L., **^{13}C spin-lattice relaxation studies of perfluoroalkanes,** *J. Am. Chem. Soc.* **98,** No. 7, 1697-1700 (Mar. 31, 1976).

16328. Oettinger, F. F., Blackburn, D. L., Rubin, S., **Thermal characterization of power transistors,** *IEEE Trans. Electron Devices* **ED-23,** No. 8, 831-838 (Aug. 1976).

16329. Sibley, E. H., **The development of data-base technology,** *Assoc. Comput. Mach. Comput. Surveys Editorial* **8,** No. 1, 1-5 (Mar. 1976).

16330. Fry, J. P., Sibley, E. H., **Evolution of data-base management systems,** *Assoc. Comput. Mach. Comput. Surveys* **8,** No. 1, 7-42 (Mar. 1976).

16331. Mielenz, K. D., **Comments on spectrometry nomenclature,** *Anal. Chem.* **48,** No. 7, 1093-1094 (June 1976 .

16332. Stevens, J. G., Travis, J. C., DeVoe, J R., **Mössbauer spectrometry,** *Anal. Chem.* **44,** No. 5, 384R-406R (Apr. 1972).

16333. Mielenz, K. D., Mavrodineanu, R., Cehelnik, E. D., **Efficient averaging spheres for visible and ultraviolet wavelengths,** *Appl. Opt.* **14,** No. 8, 1940-1947 (Aug. 1975).

16334. Kubota, M., Golightly, D. W., Mavrodineanu, R., **A rapid wire-loop method for determination of lead in paint by atomic absorption spectrometry,** *Appl. Spectrosc.* **30,** No. 1, 56-63 (1976).

16335. Lawn, B. R., **A model for the wear of brittle solids under fixed abrasive conditions,** *Wear Short Commun.* **33,** 369-372 (1975).

16336. Powell, F. J., **Trends in heating, ventilating and air-conditioning,** *Proc. 5th CIB Congress on Research into Practice. The Challenge of Application, Paris, France, June 1971,* pp. 646-648 (1971).

16337. Cassatt, W. A., Yap, W. T., Reed, W. P., Mills, S. A., **Results of measurements on fine particle standard reference materials at NBS,** *Powder Tech.* **13,** 27-31 (1976).

16338. Garbern, D., Kelly, G. E., **Dynamic efficiency of a gas-fired boiler,** *Proc. Conf. on Improving Efficiency and Performance of HVAC Equipment and Systems for Commercial and Industrial Buildings, Purdue University, Lafayette, IN, Apr. 12-14, 1976, II,* Paper C.1, pp. 330-335 (Purdue Res. Foundation, Lafayette, IN, 1976).

16339. Fong, J. T., Zapas, L. J., **Chemorheology and the mechanical behavior of vulcanized rubbers,** *Trans. Soc. Rheol.* **20:3,** 319-338 (1976).

16340. Cram, S. P., Chesler, S. N., **Coupling of high speed plasma chromatography with gas chromatography,** *J. Chromatogr. Sci.* **11,** 391-401 (Aug. 1973).

16341. Dragoo, A. L., **Steady thermocapillary convection cells in liquid drops,** *Proc. Int. Colloquium on Drops and Bubbles, Pasadena, CA, Aug. 28-30, 1974,* pp. 208-226 (1974).

16342. Kelnhofer, W. J., Hunt, C. M., Didion, D. A., **Determination of combined air exfiltration and ventilation rates in a nine-story office building,** *Proc. Conf. on Improving Efficiency and Performance of HVAC Equipment and Systems for Commercial and Industrial Buildings, Purdue University, Lafayette, IN, Apr. 12-14, 1976, II, Paper B.4,* pp. 322-328 (Purdue Res. Foundation, Lafayette, IN, 1976).

16343. Schaffer, R., **The development of a clinical reference method for glucose in serum,** *Pure Appl. Chem.* **45,** 75-79 (1976).

16344. Hertz, H. S., May, W. E., Chesler, S. N., Gump, B. H., **Analysis of μg/kg (ppb) level hydrocarbons in intertidal zone sediments and water by gas chromatography-mass spectrometry,** *Proc. 23d Annual Conf. on Mass Spectrometry and Allied Topics, Houston, TX, May 25-30, 1975,* pp. 663-665 (1975).

16345. Currie, L. A., **The measurement of environmental levels of rare gas nuclides and the treatment of very low-level counting data,** *IEEE Trans. Nucl. Sci.* **NS-19,** No. 1, 119-126 (Feb. 1972).

16346. Smith, C. N., Placious, R. C., Mills, R. M., **X-ray systems for bomb disarmament,** *NILECJ-STD-0603.00,* 11 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, June 1975).

16347. Chi, J., **Computer simulation of fossil-fuel-fired boilers,** *Proc. Conf. on Improving Efficiency and Performance of HVAC Equipment and Systems for Commercial and Industrial Buildings, Purdue University, Lafayette, IN, Apr. 12-14, 1976, II, Paper C.2,* pp. 336-346 (Purdue Res. Foundation, Lafayette, IN, 1976).

16348. Johnson, C. R., **Normality and the numerical range,** *Linear Algebra and Its Applications* **15,** 89-94 (1976).

Key words: eigenvector; normal matrix; numerical range; orthogonal; spectrum.

It is well known that if A is an n by n normal matrix, then the numerical range of A is the convex hull of its spectrum. The converse is valid for $n \leq 4$ but not for larger n. In this spirit a characterization of normal matrices is given only in terms of the numerical range. Also,

a characterization is given of matrices for which the numerical range coincides with the convex hull of the spectrum. A key observation is that the eigenvectors corresponding to any eigenvalue occurring on the boundary of the numerical range must be orthogonal to eigenvectors corresponding to all other eigenvalues.

16349. Clifton, J., Frohnsdorff, G., **Polymer-impregnated concretes,** Chapter 12. Special Review in *Cements Research Progress 1975*, pp. 173-196 (American Ceramic Society, Columbus, OH, 1976).

16350. Evans, J. M., Jr., **The role of standards in integrated manufacturing systems,** (Proc. Twelfth Annual Meeting and Tech. Conf. of the Numerical Control Society, Washington, DC, May 19-21, 1975), Paper in *Progress through Government Education and Industry Cooperation, NC/CAM EXPO 1975*, pp. 211-217 (Numerical Control Society, Inc., Spring Lake, NJ, 1975).

16351. Gerhold, W. F., McCann, J. P., Williamson, W. E., **Report on corrosion of underground telephone cable shielding materials in soil environment after exposure for four years,** *Proc. NACE Conf., Chicago, IL, Mar. 5-9, 1973*, pp. 1-33 (National Association of Corrosion Engineers, Houston, TX 77027, Mar. 1974).

16352. Gerhold, W. F., McCann, J. P., **Corrosion evaluation of underground telephone cable shielding materials,** (Proc. Int. Corrosion Forum on the Protection and Performance of Materials, Houston, TX, Mar. 22-26, 1976), *Corrosion/76*, Paper No. 31, pp. 31/1-31/62 (1976).

16353. Ogburn, F., **Metrication and the electroplaters,** *Plat. Surf. Finish*, pp. 669, 672-673 (July 1975).

16354. Escalante, E., Gerhold, W. F., **Galvanic coupling of some stressed stainless steels to dissimilar metals underground,** *Am. Soc. Test. Mater. Spec. Tech. Publ. STP* **576,** 81-93 (1976).

16355. Escalante, E., Gerhold, W. F., **The galvanic coupling of some stainless steels to copper–Underground,** (Proc. Corrosion/75, Toronto, Ontario, Canada, Apr. 14-18, 1975), *Mater. Perform.* **14,** No. 10, 16-20 (Oct. 1975).

16356. Boettinger, W. J., Burdette, H. E., Kuriyama, M., Green, R. E., Jr., **Asymmetric crystal topographic camera,** *Rev. Sci. Instrum.* **47,** No. 8, 906-911 (Aug. 1976).

16357. Segransan, P. J., Clark, W. G., Chabret, Y., Carter, G. C., **The Knight shift and nuclear spin relaxation of ^{61}Ni metal in the paramagnet phase,** *J. Phys. F: Metal Phys. Lett. to Ed.* **6,** No. 5, L153-L156 (1976).

16358. Schaffer, R., **Calibrator product class standard,** *Proc. of the SAMA Calibrator Product Class Standard Forum and Stability Testing Workshop, Chicago, IL, Mar. 18-19, 1975*, 7 pages (Scientific Apparatus Makers Association, Washington, DC 20036, 1975).

16359. Corcoran, C. T., Langhoff, P. W., **Polynomial expansions for spectral densities,** *Chem. Phys. Lett.* **41,** No. 3, 609-614 (Aug. 1, 1976).

16360. Evans, A. G., Clifton, J. R., Anderson, E., **The fracture mechanics of mortars,** *Cement Concrete Res.* **6,** No. 4, 535-547 (1976).

16361. Slater, J. A., **Data systems and fire casualty information,** *Proc. First Conf. and Workshop on Fire Casualties, Johns Hopkins University Applied Physics Laboratory, Laurel, MD, May 28-29, 1975*, pp. 158-171 (Apr. 1976).

16362. Lias, S. G., **Charge transfer to hydrogen halide and halogen molecules from atomic and diatomic ions,** *Int. J. Mass Spectrom. Ion Phys.* **20,** 123-137 (1976).

16363. Buehler, M. G., Phillips, W. E., **A study of the gold acceptor in a silicon $p+n$ junction and an n-type MOS capacitor by thermally stimulated current and capacitance measurements,** *Solid-State Electron.* **19,** 777-788 1976).

16364. Garvin, D., Hampson, R. F., Jr., **Evaluated numerical data for the SST and chlorofluorocarbon problems: A case study of how to help the engineer and the modellers,** (Proc. AGARD Conf., Washington, DC, Oct. 20-21, 1976), Paper in *AGARD Conference Preprint* **207** *on Advancement in Retrieval Technology as Related to Information Systems*, pp. 13-1–13-6 (National Aeronautics and Space Administration, Attn.: Reports Distribution and Storage Unit, Langley Field, VA 23365, 1976).

16365. Scheer, M. D., **Processes for converting coal to clean synthetic fuels and their dependence on catalytic materials,** (Proc. Ceramic Materials in High Temperature Gases on Materials Problems and Research Opportunities in Coal Conversion, Columbus, OH, Apr. 16-18, 1974), Paper in *Materials Problems and Research Opportunities in Coal Conversion* **2,** 300-312 (The Department of Metallurgical Engineering, The Ohio State University, Columbus, OH, 1975).

16366. Odell, G. B., Schaffer, R., Simopoulos, A. P., Eds., **Phototherapy in the newborn: An overview,** 196 pages (Printing and Publishing Office, National Academy of Sciences, Washington, DC, 1974).

16367. Quindry, T. L., **Sound sensing units for intrusion alarm systems,** *NILECJ-STD-0308.00*, 22 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, July 1976).

16368. Dobbyn, R. C., **Selection guide to hearing protectors for use on firing ranges,** *NILECJ-GUIDE-0101.00*, 24 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Apr. 1976).

16369. Robertson, E., Fechter, J. V., **Directory of security consultants,** *LESP-RPT-0309.00*, 64 pages (U.S. Department of Justice, Law Enforcement Assistance Administration, National Institute of Law Enforcement and Criminal Justice, Washington, DC, Oct. 1975).

16370. Kilmer, R. D., Leasure, W. A., Jr., Corley, D. M., Mathews, D. E., Shoemaker, C. O., Jr., **Truck noise 1-D: Empirical model for predicting in-service truck tire noise levels,** *Report No. DOT-TST-76T-5*, 65 pages (Available from the National Technical Information Service, Springfield, VA 22161, July 1976).

16371. Berger, H., Parker, W. L., Lapinski, N. P., Reimann, K. J., **Three-dimensional inspection by thermal-neutron laminagraphy,** *Trans. American Nuclear Society 1976 Annual Meeting, Toronto, Canada, June 14-18, 1976*, pp. 115-116 (1976).

16372. Sengers, J. M. H. L., **Critical exponents at the turn of the century,** *Physica* **82A,** 319-351 (1976).

16373. Boettinger, W. J., Burdette, H. E., Kuriyama, M., **Application of contrast conditions to dynamical images of immobile dislocations,** *Philos. Mag.* **34,** No. 1, 119-127 (1976).

16374. Kulin, G., **Discussion of "sediment transport in smooth fixed bed channels" by P. Novak and C. Nalluri,** *J. Hydraul. Div. Proc. Am. Soc. Civ. Eng.* **102,** HY10, 1601-1602 (Oct. 1976).

16375. Block, S., Piermarini, G., **The diamond cell stimulates high-pressure research,** *Phys. Today*, pp. 44-47, 50-52, 54-55 (Sept. 1976).

16376. Hougen, J. T., **Methane symmetry operations,** Chapter 3 in *International Review of Science, Spectroscopy, Physical Chemistry Series Two,* A. D. Buckingham and D. A. Ramsay, Eds., **3,** 75-125 (Butterworth & Co., Ltd., Boston, MA, 1976).

16377. Field, B. F., Hesterman, V. W., **Josephson junction based voltage standard,** *Proc. Fifth Cal Poly Measurement Science Conf., San Luis Obispo, CA, Dec. 5-6, 1975,* 5 pages (California Polytechnic State University, San Luis Obispo, CA 93407, 1975).

16378. Cooper, J. W., **The single electron model in photoionization,** (Lecture delivered at the NATO Advanced Study Institute, Carry-le-Rouet, France, Aug. 31-Sept. 13, 1975), Paper in *Photoionization and Other Probes of Many-Electron Interactions,* F. J. Wuilleumier, Ed., pp. 31-47 (Plenum Press, Inc., New York, NY, 1975).

16379. Langhoff, P. W., Corcoran, C. T., Sims, J. S., Weinhold, F., Glover, R. M., **Moment-theory investigations of photoabsorption and dispersion profiles in atoms and ions,** *Phys. Rev. A* **14,** No. 3, 1042-1056 (Sept. 1976).

16380. Hall, J. L., Lee, S. A., **Interferometric real-time display of cw dye laser wavelength with sub-Doppler accuracy,** *Appl. Phys. Lett.* **29,** No. 6, 367-369 (Sept. 15, 1976).

16381. Kahler, R. L., Hillhouse, D. L., Anderson, W. E., **A simple overcurrent protection circuit for a high voltage laboratory,** *IEEE Trans. Instrum. Meas.* **IM-25,** No. 2, 161-162 (June 1976).

16382. Hartman, A. W., **A step height interferometer with one nanometer resolution,** *Opt. Eng.* **15,** No. 2, 180-183, 363 (Mar.-Apr. 1976).

16383. Brandon, D. G., Melmed, A. J., **FIM analysis of dislocation core structure,** *J. Micros.* **100,** Pt. 1, 7 pages (Jan. 1974).

16384. Misakian, M., Hebner, R. E., Jr., **Kerr coefficients of polychlorinated biphenyls and chlorinated naphthalene,** *J. Appl. Phys.* **47,** No. 9, 4052-4055 (Sept. 1976).

16385. Goldberg, R. N., **Thermodynamics of hexokinase catalyzed reactions. II. Measurement and calculation of enthalpies of reaction as a function of magnesium ion concentration,** *Biophys. Chem.* **4,** 215-221 (1976).

16386. Cooper, J. W., Manson, S. T., **Photo-ionization in the soft x-ray range: Angular distributions of photoelectrons and interpretation in terms of subshell structure,** *Phys. Rev.* **177,** No. 1, 157-163 (Jan. 5, 1969).

16387. Swanson, K. R., Spijkerman, J. J., **Analysis of thin surface layers by Fe-57 Mössbauer backscattering spectrometry,** *J. Appl. Phys.* **41,** No. 7, 3155-3158 (June 1970).

16388. Rutherford, W. M., Evans, J., Currie, L. A., **Isotopic enrichment and pulse shape discrimination for measurement of atmospheric Argon-37,** *Anal. Chem.* **48,** No. 3, 607-612 (Mar. 1976).

16389. Shier, D. R., **Iterative methods for determining the k shortest paths in a network,** *Networks* **6,** No. 3, 205-229 (July 1976).

16390. Lowe, T. C., **Computer security safeguards for privacy: The technical role of NBS,** *Bull. Am. Soc. Inf. Sci.* **3,** No. 1, 16-18 (Oct. 1976).

16391. Johnson, C. E., Ogburn, F., **Hardness of heat treated electroless nickel,** *Surf. Technol.* **4,** 161-172 (1976).

16392. Heinrich, K. F. J., **Fluorescence excited by the continuum and the accuracy of electron probe microanalysis, (Extended Abstract),** *Proc. 10th Annual Conf. of the Microbeam Analysis Society, Las Vegas, NV, Aug. 11-15, 1975,* pp. 1A-1F (1975).

16393. Grabner, L., **Isoelectronic donor iodine and broad-band photoluminescence in TlBr,** *Phys. Rev. B* **14,** No. 6, 2514-2519 (Sept. 15, 1976).

16394. Page, C. H., **What is weight?,** *Am. J. Phys.* **43,** No. 10, 920-921 (Oct. 1975).

16395. Page, C. H., **Ambiguities in the use of unit names,** *Science* **179,** 873-875 (Mar. 2, 1973).

16396. Morrissey, B. W., Smith, L. E., Stromberg, R. R., Fenstermaker, C. A., **Ellipsometric investigation of the effect of potential on blood protein conformation and adsorbance,** *J. Colloid Interface Sci.* **56,** No. 3, 557-563 (Sept. 1976).

16397. Blevin, W. R., Steiner, B., **Redefinition of the candela and the lumen,** *Metrologia* **11,** 97-104 (1975).

16398. Madey, T. E., Czyzewski, J. J., Yates, J. T., Jr., **Ion angular distributions in electron stimulated desorption: Oxygen and CO on W(111),** *Surf. Sci.* **57,** 580-590 (1976).

16399. Layer, H. P., **High-resolution stepping motor drive,** *Rev. Sci. Instrum.* **47,** No. 4, 480-483 (Apr. 1976).

16400. Johnson, D. R., Pearson, R., Jr., **Microwave region,** Chapter 4.3 in *Methods of Experimental Physics* **13, Spectroscopy,** Part B, D. Williams, Ed., pp. 102-133 (Academic Press, Inc., New York, NY, 1976).

16401. Fiori, C. E., Myklebust, R. L., Heinrich, K. F. J., Yakowitz, H., **FRAME B: An on-line correction procedure for quantitative electron probe microanalysis with a Si(Li) detector, (Extended Abstract),** *Proc. 10th Annual Conf. of the Microbeam Analysis Society, Las Vegas, NV, Aug. 11-15, 1975,* pp. 20A-20G (1975).

16402. Pella, P. A., **Generator for producing trace vapor concentrations of 2,4,6-trinitrotoluene, 2,4-dinitrotoluene, and ethylene glycol dinitrate for calibrating explosives vapor detectors,** *Anal. Chem.* **48,** No. 11, 1632-1637 (Sept. 1976).

16403. Forester, D. W., Abbundi, R., Segnan, R., Sweger, D., **Magnetic hyperfine structure in amorphous $DyFe_2$,** (Proc. 20th Annual AIP Conf. on Magnetism and Magnetic Materials, San Francisco, CA, Dec. 3-6, 1974), Paper in *Magnetism and Magnetic Materials-1974,* C. D. Graham, Jr., G. H. Lander, and J. J. Rhyne, Eds., No. 24, 115-116 (American Institute of Physics, New York, NY, 1975).

16404. Hill, J. J., **A low frequency inductive ratio bridge for platinum resistance thermometry,** (Proc. 5th Symp. on Temperature, Its Measurement and Control in Science and Industry, Washington, DC, June 21-24, 1971), Paper in *Temperature, Its Measurement and Control in Science and Industry,* L. G. Rubin, A. C. Anderson, J. E. Janssen, and R. D. Cutkosky, Eds., **4,** Part 2, 1495-1499 (Instrument Society of America, Pittsburgh, PA, 1972).

16405. Alperin, H. A., Cullen, J. R., Clark, A. E., **Magnetic properties of bulk amorphous Tb_xFe_{1-x},** (Proc. 21st Annual AIP Conf. on Magnetism and Magnetic Materials, Philadelphia, PA, Dec. 9-12, 1975), Paper in *Magnetism and Magnetic Materials-1975,* J. J. Becker, G. H. Lander, and J. J. Rhyne, Eds., No. 29, pp. 186-187 (American Institute of Physics, New York, NY, 1976).

16406. Helbig, V., Kelleher, D. E., Wiese, W. L., **Stark-broadening study of neutral nitrogen lines,** *Phys. Rev. A* **14,** No. 3, 1082-1093 (Sept. 1976).

16407. Hall, J. L., Magyar, J. A., **High resolution saturated absorption studies of methane and some methyl-halides,** Chapter 5 in *Topics in Applied Physics* **13,** High-Resolution Laser Spectroscopy, K. Shimoda, Ed., pp. 173-199 (Springer-Verlag, New York, NY, 1976).

16408. Brown, P. W., Clifton, J. R., Frohnsdorff, G., Berger, R. L., **The utilization of industrial by-products in blended cements,** (Proc. 5th Mineral Waste Utilization Symposium, Chicago, IL, Apr. 13-14, 1976), Paper in *Proceedings of the Fifth Mineral Waste Utilization Symp.*, pp. 278-284 (ITT Research Institute, Chicago, IL, 1976).

16409. Brinckman, F. E., Iverson, W. P., Blair, W., **Approaches to the study of microbial transformations of metals,** *Proc. 3d Int. Biodegradation Symp., Kingston, RI, Aug. 17-23, 1975,* pp. 919-936 (1976).

16410. Yakowitz, H., **Present status of quantitative electron probe microanalysis,** *Proc. 6th European Congress on Electron Microscopy, Jerusalem, Israel, Sept. 1976,* pp. 37-40 (Tal International Publishing Co., Jerusalem, Israel, 1976).

16411. Simmons, J. A., **Microscopic aspects of residual stresses,** *Proc. Workshop on Nondestructive evaluation of Residual Stress, San Antonio, TX, Aug. 13-14, 1975,* pp. 11-17 (Air Force Materials Laboratory, Wright-Patterson Air Force Base, OH, 1976).

16412. MacGregor, J. G., Breen, J. E., Pfrang, E. O., **Design of slender concrete columns,** *ACI J* , pp. 6-28 (Jan. 1970 .

16413. Lind, M. A , Zalewski, E. F., **Silicon photodetector instabilities in the uv,** *Appl. Opt.* **15,** No. 6, 1377-13 8 (June 1976).

16414.. Bingham, C. D., Yolken, H. T., Reed, W. P., **Nondestructive assay measurements can be traceable,** *Nucl. Mater. Manage.* **V,** No. II, 32-35 (Summer 1976).

16415. Lucatorto, T. B., McIlrath, T. J., **Efficient laser production of a Na^+ ground-state plasma column: Absorption spectroscopy and photoionization measurement of Na^+,** *Phys. Rev. Lett.* **37,** No. 7, 428-431 (Aug. 16, 1976).

16416. Post, M. A., Iverson, W. P., Campbell, P. G., **Evaluating non-mercurial fungicides,** *Mod. Paint Coat.* **66,** No. 9, 31-38 (Sept. 1976).

16417. Grabec, I., Peterlin, A., **Acoustic emission of a crazing polymer,** *J. Polym. Sci.: Polym. Phys. Edition* **14,** 651-661 (1976).

JOURNAL OF RESEARCH of the National Bureau of Standards—B. Mathematical Sciences
Vol. 80B, No. 4, October–December 1976

INDEX TO VOLUME 80B

January–December 1976

Page

A

Algebraic numbers ... 1
Algorithm ... 337
Always convergent algorithms ... 53
Analytical function ... 403
An efficient linear algebraic algorithm for the determination of isomorphism in pairs of undirected graphs ... 447
An inequality for doubly stochastic matrices ... 433
Antenna-antenna interactions ... 5
Antenna theory ... 5
Arc-connectedness ... 75
Automatic computing methods for special functions. Part III. The sine, cosine, exponential integrals and related functions ... 291

B

Bead/Spring model ... 273
Bell Numbers ... 265
Binomial coefficients ... 99
Branching theory ... 265
Block Gaussian elimination ... 245
Buoyant flows ... 353

C

Cain, B. E., Real, 3 × 3, D-stable matrices ... 75
Carlson-Johnson conjecture ... 75
Cauchy principle value ... 403
Clebsch-Gordon series ... 265
Clique ... 441
Clique graph ... 441
Combinatorial analysis ... 237
Combinatorial probability ... 237
Complete elliptic integral ... 313
Conjugates ... 1
Continued cotangent ... 285
Continued fraction ... 285, 291
Convolution ... 347
Cooper, L., Katz, I. N., Optimal facility location for normally exponentially distributed points ... 53
Cosine integral ... 291
Cubic lattice ... 313

D

Decomposible (or pure) tensor products ... 259
Definite integral ... 313
Definite integrals of the complete elliptic integral K ... 313
Dependence of inspection-system performances on levels of penalties and inspection resources ... 189
Determinantal divisor ... 89
Diagnol scaling ... 433
Dilute polymer solutions ... 263
Doubly stochastic matrix ... 433
D-stable ... 75

E

Eigenvalue ... 273, 333
Equivalence ... 89
Equivalence of partitioned matrices ... 89
Error bounds ... 437
Exponential integral ... 291
Exponentially distributed points ... 53

Page

F

Facility location ... 53
Feinberg, R. B., Equivalence of partitioned matrices ... 89
Finding the circuit of a matroid ... 337
Fluid dynamics ... 79
Fong, J. T., Peterlin, A., Matrix algebra and eigenvalues for the Bead/Spring model of polymer solutions ... 273
FORTRAN Program ... 291
Fox, D. W., Spectral measures and separation of variables ... 347
—, Transient solutions for stratified fluid flows ... 79
Fundamental solution ... 79

G

Geophysical fluid flows ... 353
Glasser, M. L., Definite integrals of the complete elliptical integral K ... 313
Goldberg, K., Leighton, F. T., Newman, M. Zuckerman, S. L., Tables of binomial coefficients and Stirling numbers ... 99
Goldman, A. J., Pearl, M. H., The dependence of inspection-system performances on levels of penalties and inspection resources ... 189
Graph ... 245, 337
Graph isomorphism ... 447

H

Hankel matrix ... 269
Hilbert problem for two unknowns ... 403
Hilbert space ... 347
Hydrodynamic interaction ... 273
Hyperbolic sine and cosine integrals ... 291

I

Improved error bounds for second-order differential equations with two turning points ... 437
Independence ... 333
Inertial waves ... 353
Initial value problem ... 79
Inspection ... 189
Internal waves ... 353
Internal waves, buoyant flows ... 79
Intersection graph ... 441
Inversion ... 245
Inverting sparse matrices by tree partitioning ... 245
Irreducible character ... 265
Irreducible complex character ... 259

J

Jerri, A. J., Sampling expansion for a Laguerre-L^{α}_{ν} transform ... 415
Johnson, C. R., A note on pairs of matrices with product zero ... 333
—, *Kellog, R. B.*, An inequality for doubly stochastic matrices ... 433
—, *Leighton, F. T.*, An efficient linear algebraic algorithm for the determination of isomorphism in pairs of undirected graphs ... 447

K

Katz, I. N., Cooper, L., Optimal facility location for normally exponentially distributed points ... 53

Page

Kellog, R. B., Johnson, C. R., An inequality for doubly stochastic matrices ... 433
Kerns, D. M., Plane-wave scattering matrix theory of antennas and antenna-antenna interactions: formulation and applications ... 5
Key values ... 291
Kronecker power of a permutation ... 265

L

Labels ... 447
Laguerre transform ... 415
Leighton, F. T., Johnson, C. R., An efficient linear algebraic algorithm for the determination of isomorphism in pairs of undirected graphs ... 447
—, *Goldberg, K., Newman, M., Zuckerman, S. L.*, Tables of binomial coefficients and Stirling numbers ... 99

M

Mathematical model ... 189
Matric equation ... 89
Matrix algebra ... 273
Matrix algebra and eigenvalues for the Bead/Spring model of polymer solutions ... 273
Matrix functions ... 265
Matroid ... 337
Measure theory ... 237
Merris, R., The Kronecker power of a permutation ... 265
—, The structure of higher degree symmetry classes of tensors ... 259
Meyers, P. R., A minimax-measure intersection problem ... 237
Minieka, E., Finding the circuits of a matroid ... 337
Minimax ... 237
Minimax-measure intersection problem ... 237
Modified adjacent matrix ... 447
Modular coefficients ... 99
More tractable solution to a singular integral equation obtained by solving a related Hilbert problem for two unknowns ... 403
Multiple transitivity ... 265

N

Necklace model ... 273
Network ... 337
Newman, M., Non-negative sums of roots of unity ... 1
—, *Goldberg, K., Leighton, F. T., Zuckerman, S. L.*, Tables of binomial coefficients and Stirling numbers ... 99
Non-central t-distribution ... 343
Non-negative sums of roots of unity ... 1
Normal distribution ... 343
Normally distributed points ... 53
Note on pairs of matrices with product zero ... 333
Note on pseudointersection graphs ... 441
Numerical analysis ... 53

O

Olver, F. W. J., Improved error bounds for second-order differential equations with two turning points ... 437
On Iohvidov's proofs of the Fischer-Frobenius theorem ... 269
One-sided tolerance limits for the normal distribution, $P = 0.80, \gamma = 0.80$ ... 343
Operations research ... 337
Optimal facility location for normally and exponentially distributed points ... 53
Optimization ... 53
Orthogonality relations ... 259

P

Page, C. H., On the dimension group of classical Physics ... 127
Parabolic cylinder functions ... 437
Partial differential equations ... 79
Partitioned matrix ... 89
Partitioning ... 245
Pearl, M. H., Goldman, A. J., The dependence of inspection-system performance on levels of penalties and inspection resources ... 189
Pennline, J. A., A more tractable solution to a singular integral equation obtained by solving a related Hilbert problem for two unknowns ... 403
Permutation group ... 259, 265
Perron-Frobenius eigenvalue ... 433
Peterlin, A., Fong, J. T., Matrix algebra and eigenvalues for the Bead/Spring model of polymer solutions ... 273
Plane-wave scattering-matrix theory of antennas and antenna-interactions: formulation and applications ... 5
Polymer physics ... 273
Positive stable matrix ... 75
Predictable regular continued contangent expansions ... 285
Pseudointersection graph ... 441

Q

Quadratic form ... 333
Quadratic irrational ... 285

R

Random variable ... 333
Real, 3×3, D-stable matrices ... 75
Recurrence relations ... 291
Regulation ... 189
Rehm, R. G., A Survey of selected aspects of stratified and rotating fluids ... 353
Roots of unity ... 1
Rotating fluids ... 353

S

Sampling, expansion for a Laguerre-L^{α}_{ν} transform ... 415
Sampling function ... 415
Scattering-matrix theory of antennas ... 5
Separation of variables ... 347
Set covering ... 441
Shalit, J., Predictable regular continued cotangent expansions ... 285
Shier, D. R., Inverting sparse matrices by tree partitioning ... 245
Singular integral equation ... 403
Singular value decomposition ... 333
Slater, P. J., A note on pseudointersection graphs ... 441
Smith normal form ... 89
Sparse matrix ... 245
Special functions ... 313
Special measure ... 347
Spectral measures and separation of variables ... 347
Spectrum ... 447
Statistics ... 343
Stegun, I. A., Zucker, R., Automatic computing methods for special functions. Part III. The sine, cosine, exponential integrals, and related functions ... 291
Stirling numbers ... 99, 265
Stochastic Weber problem ... 53
Strategy ... 189
Stratified flow ... 79
Stratified fluids ... 353
Structure of higher degree symmetry classes of tensors ... 259
Survey of selected aspects of stratified and rotating fluids ... 353

T

Tables of binominal coefficients and Stirling numbers ... 99
Tensor products ... 347
Theory of games ... 189
Thompson, R. C., On Iohvidov's proofs of the Fischer-Frobenius theorem ... 269
Toeplitz matrix ... 269
Tolerance limits ... 343
Transient solutions for stratified fluid flows ... 79
Tree ... 245
Turning points ... 437

Page

U

Uniform asymptotic approximations 437

W

Wampler, R. H., One-sided tolerance limits for the normal distribution, P = 0.80. γ = 0.80 343
Waveguide boundary problem 403
Waves in fluids 353
Weber's equation 437
Whittaker's Cardinal function 415

Z

Zucker, R., Stegun, I. A., Automatic computing methods for special functions. Part III. The sine, cosine, exponential integrals, and related functions 291
Zuckerman, S. L., Goldberg, K., Leighton, F. T. Newman, M., Tables of binomial coefficients and Stirling numbers 99

☆ U.S. GOVERNMENT PRINTING OFFICE: 1977 O—225-726

NBS TECHNICAL PUBLICATIONS

PERIODICALS

JOURNAL OF RESEARCH reports National Bureau of Standards research and development in physics, mathematics, and chemistry. It is published in two sections, available separately:

• **Physics and Chemistry (Section A)**
Papers of interest primarily to scientists working in these fields. This section covers a broad range of physical and chemical research, with major emphasis on standards of physical measurement, fundamental constants, and properties of matter. Issued six times a year. Annual subscription: Domestic, $17.00; Foreign, $21.25.

• **Mathematical Sciences (Section B)**
Studies and compilations designed mainly for the mathematician and theoretical physicist. Topics in mathematical statistics, theory of experiment design, numerical analysis, theoretical physics and chemistry, logical design and programming of computers and computer systems. Short numerical tables. Issued quarterly. Annual subscription: Domestic, $9.00; Foreign, $11.25.

DIMENSIONS/NBS (formerly Technical News Bulletin)—This monthly magazine is published to inform scientists, engineers, businessmen, industry, teachers, students, and consumers of the latest advances in science and technology, with primary emphasis on the work at NBS. The magazine highlights and reviews such issues as energy research, fire protection, building technology, metric conversion, pollution abatement, health and safety, and consumer product performance. In addition, it reports the results of Bureau programs in measurement standards and techniques, properties of matter and materials, engineering standards and services, instrumentation, and automatic data processing.

Annual subscription: Domestic, $12.50; Foreign, $15.65.

NONPERIODICALS

Monographs—Major contributions to the technical literature on various subjects related to the Bureau's scientific and technical activities.

Handbooks—Recommended codes of engineering and industrial practice (including safety codes) developed in cooperation with interested industries, professional organizations, and regulatory bodies.

Special Publications—Include proceedings of conferences sponsored by NBS, NBS annual reports, and other special publications appropriate to this grouping such as wall charts, pocket cards, and bibliographies.

Applied Mathematics Series—Mathematical tables, manuals, and studies of special interest to physicists, engineers, chemists, biologists, mathematicians, computer programmers, and others engaged in scientific and technical work.

National Standard Reference Data Series—Provides quantitative data on the physical and chemical properties of materials, compiled from the world's literature and critically evaluated. Developed under a world-wide program coordinated by NBS. Program under authority of National Standard Data Act (Public Law 90-396).

NOTE: At present the principal publication outlet for these data is the Journal of Physical and Chemical Reference Data (JPCRD) published quarterly for NBS by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements available from ACS, 1155 Sixteenth St. N.W., Wash. D. C. 20056.

Building Science Series—Disseminates technical information developed at the Bureau on building materials, components, systems, and whole structures. The series presents research results, test methods, and performance criteria related to the structural and environmental functions and the durability and safety characteristics of building elements and systems.

Technical Notes—Studies or reports which are complete in themselves but restrictive in their treatment of a subject. Analogous to monographs but not so comprehensive in scope or definitive in treatment of the subject area. Often serve as a vehicle for final reports of work performed at NBS under the sponsorship of other government agencies.

Voluntary Product Standards—Developed under procedures published by the Department of Commerce in Part 10, Title 15, of the Code of Federal Regulations. The purpose of the standards is to establish nationally recognized requirements for products, and to provide all concerned interests with a basis for common understanding of the characteristics of the products. NBS administers this program as a supplement to the activities of the private sector standardizing organizations.

Consumer Information Series—Practical information, based on NBS research and experience, covering areas of interest to the consumer. Easily understandable language and illustrations provide useful background knowledge for shopping in today's technological marketplace.

Order **above** *NBS publications from: Superintendent of Documents, Government Printing Office, Washington, D.C. 20402.*

Order **following** *NBS publications—NBSIR's and FIPS from the National Technical Information Services, Springfield, Va. 22161.*

Federal Information Processing Standards Publications (FIPS PUBS)—Publications in this series collectively constitute the Federal Information Processing Standards Register. Register serves as the official source of information in the Federal Government regarding standards issued by NBS pursuant to the Federal Property and Administrative Services Act of 1949 as amended, Public Law 89-306 (79 Stat. 1127), and as implemented by Executive Order 11717 (38 FR 12315, dated May 11, 1973) and Part 6 of Title 15 CFR (Code of Federal Regulations).

NBS Interagency Reports (NBSIR)—A special series of interim or final reports on work performed by NBS for outside sponsors (both government and non-government). In general, initial distribution is handled by the sponsor; public distribution is by the National Technical Information Services (Springfield, Va. 22161) in paper copy or microfiche form.

BIBLIOGRAPHIC SUBSCRIPTION SERVICES

The following current-awareness and literature-survey bibliographies are issued periodically by the Bureau:

Cryogenic Data Center Current Awareness Service. A literature survey issued biweekly. Annual subscription: Domestic, $20.00; Foreign, $25.00.

Liquified Natural Gas. A literature survey issued quarterly. Annual subscription: $20.00.

Superconducting Devices and Materials. A literature survey issued quarterly. Annual subscription: $20.00. Send subscription orders and remittances for the preceding bibliographic services to National Bureau of Standards, Cryogenic Data Center (275.02) Boulder, Colorado 80302.

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards
Washington, D.C. 20234

OFFICIAL BUSINESS
Penalty for Private Use, $300

POSTAGE AND FEES PAID
EPARTMENT OF COMMERCE
COM–215
AL FOURTH-CLASS RATE
BOOK

SJRB SERIALS 0005 ISSDUE0000R I
SERIALS PROCESSING DEPT
XEROX UNIVERSITY MICROFILMS
106

JOURNAL OF RESEARCH of the National Bureau of Standards
B. Mathematical Sciences

Vol. 80B, No. 4 Oct.–Dec. 1976

Contents

Page

An inequality for doubly stochastic matrices. Charles R. Johnson and R. Bruce Kellogg. 433

Improved error bounds for second-order differential equations with two turning points. F. W. J. Olver. 437

Pseudointersection graphs. Peter J. Slater. 441

An efficient linear algebraic algorithm for the determination of isomorphism in pairs of undirected graphs. Charles R. Johnson and Frank Thomson Leighton. 447

Publications of the National Bureau of Standards. 485

Index to Volume 80B, January–December 1976 507

Library of Congress Catalog Card Number: 62–4414

For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402. Order by SD Catalog No. C 13.22/sec.B:80B (4). Price $2.25 (single copy). Subscription price: $9.00 a year; $2.25 additional for foreign mailing.

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1977